Welfare effects of housing price changes in the general equilibrium setting

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Abstract

This paper explores the aggregate welfare effects of housing price changes in the heterogeneous agent general equilibrium model with multi-sector production side. The model includes two types of households:credit-constrained households and unconstrained households. These types differ not only because of the presence or absence of credit constraints but also from the point of view of their time preference rates and factors of production which they own. The modeling of the production side of the economy is based on Davis and Heathcote (2003) and includes composite good production sector housing production sector and intermediate goods production sector. Besides welfare comparisons between steady states, the welfare changes during transition betwen steady states are also calculated

KEY WORDS : general equilibrium model, housng price changes , aggregate welfare, binding credit constraints, multi-sector production,
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1 Introduction

The US housing market over the last 13 years have been characterized by drastic changes in housing prices. In particular in the period from 1995 to 2006 according to National Association of Realtors the median family home price has increased by 190% that is has almost doubled. However, starting from 2007 because of the financial crisis and bust in the housing market, the trend has reversed and median family home price has decreased by around 100%.

Such considerable housing price shocks had substantial implications for the household consumption and welfare , which were explored in the previous literature both for individual groups of households as well as on aggregate level. For exploring the e¤ects of housing price changes on consumption and welfare of separate groups of households, life-cycle models of housing choice have been mainly used. For instance, Campbel and Cocco (2005) based on life-cycle model and UK micro-level data on real non-durable consumption growth and real housing price growth demonstrate positive correlation between increase in the growth rate of housing prices and growth rate of non-durable consumption. Li and Yao (2004) also employ life-cycle model of housing tenure choice and find that for the homeowners less than 40 years old increase in housing prices leads to welfare losses while in case of older homeowners it leads to an increase in both their welfare as well consumption. Kiyotaki and Michaelides (2007) develop an open-economy life-cycle model of a production economy where residential and commercial structures are build by using land and capital. They use the model to investigate how housing prices aggregate production and wealth distribution react to changes in technology and financial conditions and which groups of households gain and which loose from changes in fundamentals. They find that permanent increase in the growth rate of labor productivity and the decrease in the world real interest rates substantially redistribute wealth from net of houses to the net sellers with the house price hike. On average households gain from the increase in growth rate of labor productivity and loose from the decrease in the world interest rate.

Bajari et al (2005) explore aggregate welfare effects of housing price appreciation. In this paper the authors consider only exogenous change in housing price and assume that households are not credit-constrained. The authors develop a new approach to measuring the changes in consumer welfare due to changes in the prices of owner-occupied housing which defines welfare adjustment as the transfer in the form of income required to keep expected discounted utility constant, given the change in housing prices. Using their measure of welfare adjustment, the authors show that there is no change in aggregate welfare due to an increase in the price of the existing stock of housing. Tsharakyan (2007) also analyzes the effects of housing price appreciation on aggregate welfare but generalizes the previoulsy available results by incorporating credit constraints and endogenous housing price into welfare effects calculations. At first the credit constraint is incorporated into the model with endogenous housing price and it is shown that in this model housing price appreciation leads to an improvement in aggregate welfare due to the effect of credit constraint relaxation resulting from housing price appreciation. Then housing price is endogenized by modeling the supply side of the housing market. Finally the demand and supply shocks causing housing price appreciation are calibrated according to US housing market in 1995-2004 and it is demonstrated that housing price appreciation driven by given combination of demand and supply shocks still lead to improvement in aggregate welfare.

The present paper analyzes the aggregate welfare effects of housing price changes but puts more stress on modeling than Tsharakyan (2007) and Bajari et al (2005). It contributes to previous literature by building heterogeneous agent general equilibrium model in which the aggregate welfare effects of housing price changes can be studied in more comprehensive way. The model includes two types of households : credit-constrained households and unconstrained households. These types differ not only because of the presence or absence of credit constraints but also from the point of view of their time preference rates and factors of production and assets which they own . This structure insures that in equilibrium unconstrained households will lend their extra funds to credit-constrained households and credit market will clear. All the factors of production , namely capital , land and labor are owned by households and are suplied to the firms for production. There are two goods in this economy : housing and composite consumption good. The modeling of the production side of the economy is based on Davis and Heathcote (2003) and includes composite good production sector housing production sector and intermediate goods production sector.

The more explictily modeled framework allows to gain new important insights into the question of interest. First , in this model household's income and factor prices are determined endogenously so any shock causing housing price change affects also the household's income and returns on alternative investment assets such as capital bonds and housing. Moreover, in addition to the effect of housing price changes on consumption occuring through borrowing chanel (considered in Tsharakyan (2007)) , there is a direct general equilibrium effect of housing price change on consumption which explitly depends on housing price . Since the model includes several production sectors , it is possible to see how any shock causing the change in housing price leads to redistribution of factors of production between those sectors.

After the model is defined the steady state is calculated. Then it is explored what happens with aggregate welfare when different demand and supply-side shocks cause changes in housing price and economy transfers to new steady state. Sources of housing price shocks include change in interest rate, change in building permit cost , change in maximum loan-to-value ratio and changes in factor intenisities of different production sectors. Both the change of aggregate welfare in transition as well as change of aggregate welfare in the new steady state compared with the old steady state are calculated. Finally , both the effects of housing price appreciation as well as the effects of housing price decline which is currently characteristic for US housing market , are considered.

The rest of the paper is organized as follows. Section 2 describes the model. Section 3 contains derivation of steady state . Section 4 presents the welfare changes occuring as a result of each of the shocks mentioned above. Section 5 contains calibration and numerical results

2 The Model

2.1 Production sector : housing construction and composite good production

Supply side of the market is identical for both credit constrained and unconstrained versions of the model economy. The modeling the production of housing and composite good is based on Davies and Heathcote (2003). Perfectly compettitve internmediate goods producing firms allocate capital rented from the household and labor supplied by the households between production of two intermediate goods namely manufactures and services which are indexed correspondingly by $j \in \{m, s\}$. All intermediate goods are produced using Cobb-Douglass technology but the capital's and labor's shares of income are calibrated so as to take into account the differences in factor intensivness of different intermediate goods . Each intermediate good production sector is subject to common productivity shock denoted by z_t . The production function is given by $Y_{j,t} = z_t K_{j,t}^{\alpha_j} N_{j,t}^{1-\alpha_j}$ $j \in \{m, s\}$. The maximization problem for intermediate goods producer is given by:

$$Max_{\{K_{j,t},N_{j,t}\}_{j\in\{m,s\}}} \sum_j \{p_{j,t} z_t K_{j,t}^{\alpha_j} N_{j,t}^{1-\alpha_j} - w_t N_{j,t} - r_t K_{j,t}\}$$

$$\text{s.t. } \{K_{j,t}, N_{j,t}\} \geq 0$$

$$\log z_t = a \log \overline{\overline{z}} + (1-a) \log z_{t-1} + \xi_t$$

where $p_{j,t}$ is the price of good j.The profit maximazing conditions for intermediate good producing firms are given by :

$$p_{j,t} \alpha_j z_t K_{j,t}^{\alpha_j - 1} N_{j,t}^{1-\alpha_j} = r_t \quad \text{for } j \in \{m, s\} \tag{1}$$

$$p_{j,t}(1-\alpha_j) z_t K_{j,t}^{\alpha_j} N_{j,t}^{-\alpha_j} = w_t \text{ for } j \in \{m, s\} \tag{2}$$

The goods produced by intermediate good producers are used as inputs by final good producers to produce composite consumption good and residential investment good . Let's index by c the consumption good and by res the residential investment good . The production function for final good $d\ \epsilon\{c, res\}$ is given by $Y_{d,t} = X_{m,d,t}^{\alpha_d} X_{s,d,t}^{1-\alpha_d}$ where $X_{m,d,t}$ and $X_{s,d,t}$ denote quantities of correspondingly manufactures and services used in production of final good d Consumption good producer's problem is given by :

$$Max_{\{X_{m,c,t}X_{s,c,t}\}}\{X_{m,c,t}^{\alpha_c} X_{s,c,t}^{1-\alpha_c} - p_{m,t}X_{m,c,t} - p_{s,t}X_{s,c,t}\}$$

$$s.t.\{X_{m,c,t}, X_{s,t}\} \geq 0$$

F.O.Cs for this problem are given by:

$$\alpha_c X_{m,c,t}^{\alpha_c-1} X_{s,c,t}^{1-\alpha_c} = p_{m,t} \tag{3}$$

$$(1-\alpha_c) X_{m,c,t}^{\alpha_c} X_{s,c,t}^{-\alpha_c} = p_{s,t} \tag{4}$$

Residential good production on the other hand is specific because besides the intermediate goods it requires additional building materials (bricks , concrete etc) and also it implies the costs associated with acquiring permits from regalutory authorities . The additional residential invetsment good production costs per unit of output are denoted η_t. These additional costs are exogenous state variable and source of supply shock on the housing market , the process for which is based on materials component of construction cost index. The residential good producer's problem is given by:

$$Max_{\{X_{m,res,t},X_{s,res,t}\}}\{p_{res,t}X_{m,res,t}^{\alpha_{res}} X_{s,res,t}^{1-\alpha_{res}} - p_{m,t}X_{m,res,t} -$$

$$-p_{s,t}X_{s,res,t}\ -\eta_t X_{m,res,t}^{\alpha_{res}} X_{s,res,t}^{1-\alpha_{res}}\}$$

$$\text{s.t. } \{X_{m,c,t}, X_{m,res,t}, X_{s,c,t}, X_{s,res,t}\} \geq 0$$

$$\eta_t = \sigma + \rho\eta_{t-1} + \varsigma_t$$

Residential good producer's optimization conditions are given by :

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$$p_{res,t}\alpha_{res}X_{m,res,t}^{\alpha_{res}-1}X_{s,res,t}^{1-\alpha_{res}} - \eta_t\alpha_{res}X_{m,res,t}^{\alpha_{res}-1}X_{s,res,t}^{1-\alpha_{res}} = p_{m,t} \tag{5}$$

$$p_{res,t}(1-\alpha_{res})X_{m,res,t}^{\alpha_{res}}X_{s,res,t}^{-\alpha_{res}} - \eta_t(1-\alpha_{res})X_{m,res,t}^{\alpha_{res}}X_{s,res,t}^{-\alpha_{res}} = p_{s,t} \tag{6}$$

Finally, housing construction firms combine residential investment good with land supplied by the households to produce housing units. The production function for housing is given by $F_t = (X_{res,t})^{\epsilon}(La_t)^{1-\varepsilon}$, where $X_{res,t}$ stands for amount of residential investment good used in production of housing units and La_t stands for amount of land used.

The profit maximization of construction firm is thus given by :

$$Max_{\{X_{res,t},La_t\}}\{q_t(X_{res,t})^{\varepsilon}(La_t)^{1-\varepsilon} - p_{res,t}X_{res,t} - p_{l,t}La_t\}$$

$$\{X_{res,t}, La_t\} \geq 0$$

where q_t stands for the price of a housing unit and $p_{l,t}$ stands for the price of land . The profit maximizing conditions for housing construction firms are given by:

$$q_t\varepsilon X_{res,t}^{\varepsilon-1}{La_t}^{1-\varepsilon} = p_{res,t} \tag{7}$$

$$q_t(1-\varepsilon)X_{res,t}^{\varepsilon}{La_t}^{-\varepsilon} = p_{l,t} \tag{8}$$

2.2 Households

There are two types of households in the model ,namely credit constrained households with population of size 1 and unconstrained households with population of size g. The most important differences between these types is, correspondingly, the presence and absence of credit constraints in their optimization problems. In addition to ensure that in equilibrium unconstrained households will lend funds to constrained ones I assume the different structure of owned factors of production and different rates of time preference for each of the types.

Constrained households own part of the land in the economy which they supply to the construction firms. It is assumed that each household own certain fixed amount of land $\overline{L}_c$ which can calibrated later from US data. They also supply labor to the composite good producing firms. Here I consider inelastic labor supply case and normalize it to 1 , since labor choice is not important for my analysis. Households derive utility from consumption of housing and composite consumption good and their preferences are denoted by $u(c_{c,t}, h_{c,t})$. Constrained households can invest into risk-free bonds and if the bond holdings chosen by them are negative it means that households are borrowers.The realtive price of bonds in terms of consumption good is denoted by $p_{b,t}$. It is assumed that credit-constrained households have higher impatience so their discount factor is lower. The discount factor of credit constrained households is denoted by β^c Constrained households are subject to credit constraint of the form $b_{c,t+1} \geq -m_t q_t h_{c,t+1}$ implying that each period households can borrow only certain fraction m_t of the current value of their housing. Fraction m_t is stocastic and evolves over time according to 3 state Markov process with values m_l, m_m and m_h. The first value can be though of as corresponding to situation before rapid liberalization of mortgage markets before the end of 1990s . Third value corresponds to a rapid liberalization of credit market which happened in at the end of 1990s and continued untill 2005 . The middle value corresponds to contraction of loan to value ratios after the beginning of recent financial crisis. Housing depreciates at constant rate δ_h. In what follows the allocations chozen by credit-constrained households are distinguished by subscript c . Households choose how many bonds to carry into next period $b_{c,t+1}$, how much housing to carry into next period $h_{c,t+1}$,how much to consume now $c_{c,t}$. preferences of the households

Based on the assumptions above the constrained household problem can be formulated as follows:

$$V_c(h_{c,t}, b_{c,t}, \eta_t, z_t, m_t) = \max_{\{c_{c,t}, h_{c,t+1}, b_{c,t+1}\}} \{u(c_{c,t}, h_{c,t}) + \beta^c E_t V_c(h_{c,t+1}, b_{c,t+1}, \eta_{t+1}, z_{t+1}, m_{t+1})\} \tag{9}$$

s.t.

$$c_{c,t} + q_t x_{c,t} + p_{b,t} s_{c,t} = w_t + p_{l,t}\overline{L_c} + i_t b_{c,t} \tag{10}$$

$$b_{c,t+1} - b_{c,t} = s_{c,t} \tag{11}$$

$$h_{c,t+1} - h_{c,t} = x_{c,t} - \delta_h h_{c,t} \tag{12}$$

$$b_{c,t+1} \geq -m_t q_t h_{c,t+1} \tag{13}$$

Taking F.O.C. ,rearranging, and using utlity function of the form $u(c,h) = \frac{c^{1-\gamma}}{1-\gamma} + \theta\frac{h^{1-\gamma}}{1-\gamma}$ based on Campbell and Cocco (2004) yields the following Euler equations for credit-constrained households:

$$\upsilon_t = p_{b,t} c_{c,t}^{-\gamma} - \beta^c E_t c_{c,t+1}^{-\gamma}(i_{t+1} + p_{b,t+1}) \tag{14}$$

$$q_t c_{c,t}^{-\gamma} = \beta^c \theta h_{c,t+1}^{-\gamma} + \beta^c E_t c_{c,t+1}^{-\gamma} q_{t+1}(1-\delta_h) + \\ + m_t q_t \upsilon_t \quad (15)$$

where υ_t is the multiplier of credit constraint .

The unconstrained households also possess certain amount of land which is denoted by $\overline{L}_{uc}$. Each of them supplies one unit of labor to th intermediate good producers. In addtion, unconstrained households own capital, which they supply to the intermediate good producers. Providing additional sources of income to the unconstrained households is on one hand justified from the modeling perspective by ensuring that they have additional wealth to lend in the equilibrium and on the other hand by the fact that in real life unconstrained households usually have higher net worth which is the value of wealth minus liabilities, than constrained households. Capital depreciates at rate δ_k. Investment of unconstrained households into capital denoted by in_t.The allocations made by unconstrained households are denoted by subscript uc.To ensure that unconstrained households have incentives to lend It is assumed that unconstrained households have low impatience so their discount factor is high. The discount factor of credit constrained households is denoted by β^{uc}. Unconstrained households choose how many bonds to carry into next period $b_{uc,t+1}$, how much housing to carry into next period $h_{uc,t+1}$,how much to consume now $c_{uc,t}$ and how much capital to carry into next period $h_{uc,t+1}$. The optimization problem of unconstrained households is given by:

$$V_{uc}(h_{uc,t}, b_{uc,t}, k_t, \eta_t, z_t, m_t) = \max_{\{c_{uc,t}, h_{uc,t+1}, b_{uc,t+1}, k_{t+1}\}} \{u(c_{uc,t}, h_{uc,t}) + \beta^{uc} E_t V_{uc}(h_{uc,t+1}, b_{uc,t+1}, k_{t+1}, \eta_{t+1}, z_{t+1}, m_{t+1} \quad (16)$$

s.t.

$$c_{uc,t} + q_t x_{uc,t} + p_{b,t} s_{uc,t} + in_t = w_t + p_{l,t}\overline{L_{uc}} + i_t b_{uc,t} + r_t k_t \tag{17}$$

$$b_{uc,t+1} - b_{uc,t} = s_{uc,t} \tag{18}$$

$$h_{uc,t+1} - h_{uc,t} = x_{uc,t} - \delta_h h_{uc,t} \tag{19}$$

$$k_{t+1} - k_t = in_t - \delta_k k_t \tag{20}$$

Taking F.O.C. ,rearranging, and using utlity function of the form $u(c,h) = \frac{c^{1-\gamma}}{1-\gamma} + \theta\frac{h^{1-\gamma}}{1-\gamma}$ based on Campbell and Cocco (2004) yields the following Euler equations for unconstrained households:

$$p_{b,t} c_{uc,t}^{-\gamma} = \beta^{uc} E_t c_{uc,t+1}^{-\gamma}(i_{t+1} + p_{b,t+1}) \tag{21}$$

$$q_t c_{uc,t}^{-\gamma} = \beta^{uc}\theta E_t h_{uc,t+1}^{-\gamma} + \beta^{uc} E_t c_{uc,t+1}^{-\gamma} q_{t+1}(1-\delta_h) \tag{22}$$

$$c_{uc,t}^{-\gamma} = \beta^{uc} E_t c_{uc,t+1}^{-\gamma}(1 + r_{t+1} - \delta_k) \tag{23}$$

2.3 Definition of equilibrium

The equilibrium consists of prices $\{q_t, r_t, w_t, p_{res,t}, p_{m,t}, p_{s,t}, p_{l,t}, p_{b,t}\}_{t=0}^{\infty}$, , shadow price of credit constraint $\{\upsilon_t\}_{t=0}^{\infty}$ interest rate $\{i_t\}_{t=0}^{\infty}$, allocations $\{c_{c,t}, h_{c,t+1}, b_{c,t+1}, c_{uc,t}, h_{uc,t+1}, b_{uc,t+1}, k_{t+1}\}_{t=0}^{\infty}$ by households and the profit maximizing input demands of firms $\{K_{m,t}, K_{s,t}, L_{s,t}, L_{m,t}, La_t, X_{m,c,t}, X_{m,res,t}, X$ such that:

1) given prices, households solve their optimization problem (conditions (14)-(15) and (21)-(23)) and firms maximize their profits (conditions (1)-(8))

2) Markets clear

i) $x_{c,t} + gx_{uc,t} = F_t$ (housing market)

ii) $Y_{c,t} = c_{c,t} + gc_{uc,t}$ (composite good market)

iii) $K_{m,t} + K_{s,t} = gk_{t+1}$ (capital market)

iv) $b_{c,t+1} = -\ gb_{uc,t+1}$ (credit market)

v) $N_{m,t} + N_{s,t} = g + 1$ (labor market)

vi) $X_{m,c,t} + X_{m,res,t} = Y_{m,t}$ (intermediate good market)

vii) $X_{s,c,t} + X_{s,res,t} = Y_{s,t}$ (intermediate good market)

viii) $X_{res,t} = Y_{res,t}$ (residential investnment good market)

ix) $La_t = \overline{L_c} + g\overline{L_{uc}}$ (land market)

3 Steady State

In what follows I consider the situation in which credit constraint binds , which is justified in the introduction. In terms of the model this assumption implies that following should hold:

$$\frac{p_{b,t}}{(i_{t+1} + p_{b,t+1})} < \frac{c_{c,t}^{-\gamma}}{\beta^c c_{c,t+1}^{-\gamma}}$$

In other words intertemporal MRS of credit constrainded households shold be higher than real return on bonds

Given the assumption of binding credit constraint, the steady state satisfies the following conditions:

$h_{c,t+1} = h_{c,t} = h_c^{ss}$

$$h_{uc,t+1} = h_{uc,t} = h_{uc}^{ss}$$

$$c_{c,t+1} = c_{c,t} = c_c^{ss}$$

$$c_{uc,t+1} = c_{uc,t} = c_{uc}^{ss}$$

$$b_{c,t+1} = b_{c,t} = b_c^{ss}$$

$$b_{uc,t+1} = b_{uc,t} = b_{uc}^{ss}$$

$$k_{t+1} = k_t = k^{ss}$$

$$s_c^{ss} = b_c^{ss} - b_c^{ss} = 0$$

$$s_c^{ss} = b_{uc}^{ss} - b_{uc}^{ss} = 0$$

$$x_c^{ss} = h_c^{ss} - (1 - \delta_h)h_c^{ss} = \delta_h h_c^{ss}$$

$$x_{uc}^{ss} = h_{uc}^{ss} - (1 - \delta_h)h_{uc}^{ss} = \delta_h h_{uc}^{ss}$$

$$in_{ss} = k^{ss} - (1 - \delta_k)k^{ss} = \delta_k k^{ss}$$

Rewriting binding credit constraint and credit market equilibrium condition in steady state yields the following expressions for b_c^{ss} and b_{uc}^{ss}:

$$b_c^{ss} = -m^{ss} q^{ss} h_c^{ss} \tag{24}$$

$$b_{uc}^{ss} = -\frac{b_c^{ss}}{g} \tag{25}$$

Using the above conditions budgets constraints for the constrained and unconstrained households in the steady state can be rewritten as:

$$c_c^{ss} = w^{ss} + p_l^{ss}\overline{L}_c + i^{ss} b_c^{ss} - \delta_h q^{ss} h_c^{ss} \tag{26}$$

$$c_{uc}^{ss} = w^{ss} + p_l^{ss}\overline{L}_{uc} + (r^{ss} - \delta_k)k^{ss} + i^{ss} b_{uc}^{ss} - \delta_h q^{ss} h_{uc}^{ss} \tag{27}$$

Conditions (14)-(15) for the credit constrained households in the steady state can be rewritten as :

$$\upsilon^{ss} = p_b^{ss}(c_c^{ss})^{-\gamma} - \beta^c(c_c^{ss})^{-\gamma}(i^{ss} + p_b^{ss}) \tag{28}$$

$$q^{ss} = \beta^c \frac{\theta(h_c^{ss})^{-\gamma}}{(c_c^{ss})^{-\gamma}} + \beta^c q^{ss}(1-\delta_h) + m^{ss}q^{ss}(p_b^{ss} - \beta^c(i^{ss} + p_b^{ss})) \tag{29}$$

Conditions (21)-(23) for unconstraned households in the steady state are given by the following:

$$p_b^{ss} = \beta^{uc}(i^{ss} + p_b^{ss}) \tag{30}$$

$$q^{ss} = \beta^{uc} \frac{\theta(h_{uc}^{ss})^{-\gamma}}{(c_{uc}^{ss})^{-\gamma}} + \beta^{uc} q^{ss}(1-\delta_h) \tag{31}$$

$$1 = \beta^{uc}(1 + r^{ss} - \delta_k) \tag{32}$$

Rearranging equation (25) yields :

$$c_c^{ss} = \left(\frac{1 - \beta^c(1-\delta_h) + m\beta^c i^{ss} - m p_b^{ss}(1-\beta^c)}{\beta^c \theta} q^{ss} \right)^{1/\gamma} h_c^{ss} \tag{33}$$

Rearranging (30) yields:

$$p_b^{ss} = \frac{\beta^{uc}}{1-\beta^{uc}} i^{ss} \tag{34}$$

Rearranging (31) yields:

$$c_{uc}^{ss} = \left(\frac{1 - \beta^{uc}(1-\delta_h)}{\beta^{uc}\theta} q^{ss} \right)^{1/\gamma} h_{uc}^{ss} \tag{35}$$

Rearranging (32) yields:

$$r^{ss} = \frac{1}{\beta^{uc}} - 1 + \delta_k \tag{36}$$

The steady state level of capital and the rest of the prices can be determined by solving supply side of the model and using market clearing conditions. Rewriting the conditions (1)-(2) in the steady state yields :

$$p_m^{ss} z^{ss} (K_m^{ss})^{\alpha_m - 1} (N_m^{ss})^{1-\alpha_m} = r^{ss} \tag{37}$$

$$p_m^{ss} z^{ss} (1 - \alpha_m)(K_m^{ss})^{\alpha_m} (N_m^{ss})^{-\alpha_m} = w^{ss} \tag{38}$$

$$p_s^{ss} z^{ss} (K_s^{ss})^{\alpha_s - 1} (N_s^{ss})^{1-\alpha_s} = r^{ss} \tag{39}$$

$$p_s^{ss} z^{ss} (1 - \alpha_s)(K_s^{ss})^{\alpha_s} (N_s^{ss})^{-\alpha_s} = w^{ss} \tag{40}$$

Let's assume that in steady state fraction ψ of total capital is allocated to production of manufactures and fraction $1 - \psi$ is allocated to production of services. This implies that in the steady state $K_m^{ss} = \psi k^{ss}$ and $K_s^{ss} = (1 - \psi)k^{ss}$ Similarly, fraction ϕ of total labor is allocated to production of manufactures and fraction $1 - \phi$ is allocated to production of services. Since in equibrium total labor supply is equal to $1+g$, it is true that $N_m^{ss} = \phi(1+g)$ and $N_s^{ss} = (1 - \phi)(1 + g)$. Substituting this into conditions (37)- (40) yields :

$$p_m^{ss} \alpha_m (\psi k^{ss})^{\alpha_m - 1} (\phi(1 + g))^{1-\alpha_m} = r^{ss} \tag{41}$$

$$p_m^{ss} (1 - \alpha_m)(\psi k^{ss})^{\alpha_m} (\phi(1 + g))^{-\alpha_m} = w^{ss} \tag{42}$$

$$p_s^{ss}\alpha_s((1-\psi)k^{ss})^{\alpha_s-1}((1-\phi)(1+g))^{1-\alpha_s}=r^{ss} \tag{43}$$

$$p_s^{ss}(1-\alpha_s)((1-\psi)k^{ss})^{\alpha_s}((1-\phi)(1+g))^{-\alpha_s}=w^{ss} \tag{44}$$

Dividing equation (41) by equation (42) and equation (43) by equation (44) and rearranging yields:

$\phi = \frac{\alpha_s\psi-\alpha_m\alpha_s\psi}{\alpha_m-\alpha_m\psi-\alpha_m\alpha_s+\alpha_s\psi}$

Substituting the last equation into condition (41) and condition (43), equating the resulting equations and solving for k^{ss} yields:

$$k^{ss}=\left(\frac{p_s^{ss}(1-\alpha_s)^{1-\alpha_s}\alpha_s^{\alpha_m}}{p_m^{ss}(1-\alpha_m)^{1-\alpha_m}\alpha_m^{\alpha_s}}\right)^{\frac{1}{\alpha_m-\alpha_s}}\frac{1+g}{\alpha_m-\alpha_m\psi-\alpha_m\alpha_s+\alpha_s\psi} \tag{45}$$

Using the conditions (37) and (39) it is possible to write:

$$K_m^{ss}=\left(\frac{p_m^{ss}\alpha_m}{r^{ss}}\right)^{\frac{1}{1-\alpha_m}}\phi(1+g)$$

$$K_s^{ss}=\left(\frac{p_s^{ss}\alpha_s}{r^{ss}}\right)^{\frac{1}{1-\alpha_s}}(1-\phi)(1+g)$$

Now using capital market clearing condition it is possible to derive ψ :

$$\left(\frac{p_m^{ss}\alpha_m}{r^{ss}}\right)^{\frac{1}{1-\alpha_m}}\left(\frac{\alpha_s\psi-\alpha_m\alpha_s\psi}{\alpha_m-\alpha_m\psi-\alpha_m\alpha_s+\alpha_s\psi}\right)(1+g)+\left(\frac{p_s^{ss}\alpha_s}{r^{ss}}\right)^{\frac{1}{1-\alpha_s}}\left(1-\frac{\alpha_s\psi-\alpha_m\alpha_s\psi}{\alpha_m-\alpha_m\psi-\alpha_m\alpha_s+\alpha_s\psi}\right)(1+g)=$$
$$=g\left(\frac{p_s^{ss}(1-\alpha_s)^{1-\alpha_s}\alpha_s^{\alpha_m}}{p_m^{ss}(1-\alpha_m)^{1-\alpha_m}\alpha_m^{\alpha_s}}\right)^{\frac{1}{\alpha_m-\alpha_s}}\frac{1+g}{\alpha_m-\alpha_m\psi-\alpha_m\alpha_s+\alpha_s\psi}$$

Solving the equation above for ψ yields :

$$\psi=\frac{g\left(\frac{p_s^{ss}(1-\alpha_s)^{1-\alpha_s}\alpha_s^{\alpha_m}}{p_m^{ss}(1-\alpha_m)^{1-\alpha_m}\alpha_m^{\alpha_s}}\right)^{\frac{1}{\alpha_m-\alpha_s}}-\left(\frac{p_s^{ss}\alpha_s}{r^{ss}}\right)^{\frac{1}{1-\alpha_s}}\alpha_m(1-\alpha_s)}{\left(\frac{p_m^{ss}\alpha_m}{r^{ss}}\right)^{\frac{1}{1-\alpha_m}}\alpha_s(1-\alpha_m)-\left(\frac{p_s^{ss}\alpha_s}{r^{ss}}\right)^{\frac{1}{1-\alpha_s}}\alpha_m(1-\alpha_s)}$$

To make it simpler I define some constants . Suppose that $A=g\left(\frac{(1-\alpha_s)^{1-\alpha_s}\alpha_s^{\alpha_m}}{(1-\alpha_m)^{1-\alpha_m}\alpha_m^{\alpha_s}}\right)^{\frac{1}{\alpha_m-\alpha_s}}$, $D=\left(\frac{\alpha_s}{r^{ss}}\right)^{\frac{1}{1-\alpha_s}}\alpha_m(1-\alpha_s)$, $E=\left(\frac{\alpha_m}{r^{ss}}\right)^{\frac{1}{1-\alpha_m}}\alpha_s(1-\alpha_m)$. Then the equilibrium shares can be rewritten as follows:

$$\psi = \frac{A(p_s^{ss})^{\frac{1}{\alpha_m-\alpha_s}} - D(p_s^{ss})^{\frac{1}{1-\alpha_s}}(p_m^{ss})^{\frac{1}{\alpha_m-\alpha_s}}}{E(p_m^{ss})^{\frac{1-\alpha_s}{(1-\alpha_m)(\alpha_m-\alpha_s)}} - D(p_s^{ss})^{\frac{1}{1-\alpha_s}}(p_m^{ss})^{\frac{1}{\alpha_m-\alpha_s}}} \tag{46}$$

$$\phi = \frac{(\alpha_s - \alpha_m\alpha_s)\left(A(p_s^{ss})^{\frac{1}{\alpha_m-\alpha_s}} - D(p_s^{ss})^{\frac{1}{1-\alpha_s}}(p_m^{ss})^{\frac{1}{\alpha_m-\alpha_s}}\right)}{\alpha_m(1-\alpha_s)E(p_m^{ss})^{\frac{1-\alpha_s}{(1-\alpha_m)(\alpha_m-\alpha_s)}} - \alpha_s(1-\alpha_m)D(p_s^{ss})^{\frac{1}{1-\alpha_s}}(p_m^{ss})^{\frac{1}{\alpha_m-\alpha_s}} - (\alpha_m-\alpha_s)A(p_s^{ss})^{\frac{1}{\alpha_m-\alpha_s}}} \tag{47}$$

Production volumes of intermediate good in the steady state are given by:

$$Y_m^{ss} = z^{ss}\left(\frac{p_m^{ss}\alpha_m}{r^{ss}}\right)^{\frac{\alpha_m}{1-\alpha_m}} \frac{(1+g)(\alpha_s-\alpha_m\alpha_s)\left(A(p_s^{ss})^{\frac{1}{\alpha_m-\alpha_s}} - D(p_s^{ss})^{\frac{1}{1-\alpha_s}}(p_m^{ss})^{\frac{1}{\alpha_m-\alpha_s}}\right)}{\alpha_m(1-\alpha_s)E(p_m^{ss})^{\frac{1-\alpha_s}{(1-\alpha_m)(\alpha_m-\alpha_s)}} - \alpha_s(1-\alpha_m)D(p_s^{ss})^{\frac{1}{1-\alpha_s}}(p_m^{ss})^{\frac{1}{\alpha_m-\alpha_s}} - (\alpha_m-\alpha_s)A(p_s^{ss})^{\frac{1}{\alpha_m-\alpha_s}}}$$

$$Y_s^{ss} = z^{ss}\left(\frac{p_s^{ss}\alpha_s}{r^{ss}}\right)^{\frac{\alpha_s}{1-\alpha_s}} \frac{(1+g)(\alpha_m-\alpha_m\alpha_s)\left(E(p_m^{ss})^{\frac{1-\alpha_s}{(1-\alpha_m)(\alpha_m-\alpha_s)}} - A(p_s^{ss})^{\frac{1}{\alpha_m-\alpha_s}}\right)}{\alpha_m(1-\alpha_s)E(p_m^{ss})^{\frac{1-\alpha_s}{(1-\alpha_m)(\alpha_m-\alpha_s)}} - \alpha_s(1-\alpha_m)D(p_s^{ss})^{\frac{1}{1-\alpha_s}}(p_m^{ss})^{\frac{1}{\alpha_m-\alpha_s}} - (\alpha_m-\alpha_s)A(p_s^{ss})^{\frac{1}{\alpha_m-\alpha_s}}}$$

Now let's use the profit maximization conditions of final good producers. In the steady they are given by :

$$\alpha_c(X_{m,c}^{ss})^{\alpha_c-1}(X_{s,c}^{ss})^{1-\alpha_c} = p_m^{ss} \tag{48}$$

$$(1-\alpha_c)(X_{m,c}^{ss})^{\alpha_c}(X_{s,c}^{ss})^{-\alpha_c} = p_s^{ss} \tag{49}$$

$$p_{res}^{ss}\alpha_{res}(X_{m,res}^{ss})^{\alpha_{res}-1}(X_{s,res}^{ss})^{1-\alpha_{res}} - \eta^{ss}\alpha_{res}(X_{m,res}^{ss})^{\alpha_{res}-1}(X_{s,res}^{ss})^{1-\alpha_{res}} = p_m^{ss} \tag{50}$$

$$p_{res}^{ss}(1-\alpha_{res})(X_{m,res}^{ss})^{\alpha_{res}}(X_{s,res}^{ss})^{-\alpha_{res}} - \eta^{ss}(1-\alpha_{res})(X_{m,res}^{ss})^{\alpha_{res}}(X_{s,res}^{ss})^{-\alpha_{res}} = p_s^{ss} \tag{51}$$

Dividing (48) by (49) and (50) by (51) yields:

$$X_{m,c}^{ss} = \frac{p_s^{ss}\alpha_c}{p_m^{ss}(1-\alpha_c)}X_{s,c}^{ss}$$

$$X_{s,res}^{ss} = \frac{p_m^{ss}(1-\alpha_{res})}{p_s^{ss}\alpha_{res}}X_{m,res}^{ss}$$

Now we can use intermediate good market clearing condition:

$X_{m,c}^{ss} + X_{m,res}^{ss} = Y_m^{ss}$

$X_{m,c}^{ss} = Y_m^{ss} - X_{m,res}^{ss}$

$\frac{p_s^{ss}\alpha_c}{p_m^{ss}(1-\alpha_c)} X_{s,c}^{ss} = Y_m^{ss} - X_{m,res}^{ss}$

$X_{s,c}^{ss} = \frac{p_m^{ss}(1-\alpha_c)}{p_s^{ss}\alpha_c}(Y_m^{ss} - X_{m,res}^{ss})$

$X_{s,c}^{ss} + X_{s,res}^{ss} = Y_s^{ss}$

$\frac{p_m^{ss}(1-\alpha_c)}{p_s^{ss}\alpha_c}(Y_m^{ss} - \frac{p_s^{ss}\alpha_{res}}{p_m^{ss}(1-\alpha_{res})} X_{s,res}^{ss}) + X_{s,res}^{ss} = Y_s^{ss}$

$X_{s,res}^{ss} = \frac{Y_s^{ss} - \frac{p_m^{ss}(1-\alpha_c)}{p_s^{ss}\alpha_c} Y_m^{ss}}{1 - \frac{(1-\alpha_c)\alpha_{res}}{(1-\alpha_{res})\alpha_c}}$

Substituting expressions for Y_s^{ss} and Y_m^{ss} into the last equation and simplifying yields:

$$X_{s,res}^{ss} = \frac{(G+U)((p_m^{ss})^{\alpha_1}(p_s^{ss})^{\alpha_3} - T(p_s^{ss})^{\alpha_3}(p_m^{ss})^{\frac{1}{1-\alpha_m}} - M(p_s^{ss})^{\alpha_2}}{\frac{\alpha_c - \alpha_{res}}{(1-\alpha_{res})} p_s^{ss} \left(\alpha_m(1-\alpha_s)E(p_m^{ss})^{\alpha_1} - \alpha_s(1-\alpha_m)D(p_s^{ss})^{\frac{1}{1-\alpha_s}}(p_m^{ss})^{\alpha_3} - (\alpha_m - \alpha_s)A(p_s^{ss})^{\alpha_3} \right)} \tag{52}$$

where $G = (1+g)\left(\frac{\alpha_s}{r^{ss}}\right)^{\frac{\alpha_s}{1-\alpha_s}} \alpha_c(\alpha_m - \alpha_m\alpha_s)E,\ M = (1+g)\left(\frac{\alpha_s}{r^{ss}}\right)^{\frac{\alpha_s}{1-\alpha_s}} \alpha_c(\alpha_m - \alpha_m\alpha_s)A,$
$T = (1+g)(1-\alpha_c)\left(\frac{\alpha_m}{r^{ss}}\right)^{\frac{\alpha_m}{1-\alpha_m}} (\alpha_m - \alpha_m\alpha_s)A,\ U = (1+g)(1-\alpha_c)\left(\frac{\alpha_m}{r^{ss}}\right)^{\frac{\alpha_m}{1-\alpha_m}} (\alpha_m - \alpha_m\alpha_s)D,$
$\alpha_1 = \frac{1-\alpha_s}{(1-\alpha_m)(\alpha_m - \alpha_s)},\ \alpha_2 = \frac{1+\alpha_m - 2\alpha_s}{(1-\alpha_s)(\alpha_m - \alpha_s)},\ \alpha_3 = \frac{1}{(\alpha_m - \alpha_s)}$

Substituting back yields expression for $X_{m,res}^{ss}$

$$X_{m,res}^{ss} = \frac{\alpha_{res}}{p_m^{ss}(\alpha_c - \alpha_{res})} \frac{(G+U)(p_m^{ss})^{\alpha_1}(p_s^{ss})^{\alpha_3} - T(p_s^{ss})^{\alpha_3}(p_m^{ss})^{\frac{1}{1-\alpha_m}} - M(p_s^{ss})^{\alpha_2}}{\left(\alpha_m(1-\alpha_s)E(p_m^{ss})^{\alpha_1} - \alpha_s(1-\alpha_m)D(p_s^{ss})^{\frac{1}{1-\alpha_s}}(p_m^{ss})^{\alpha_3} - (\alpha_m - \alpha_s)A(p_s^{ss})^{\alpha_3} \right)} \tag{53}$$

Using substituting the last equation into equation $X_{m,c}^{ss} = Y_m^{ss} - X_{m,res}^{ss}$ yields the expression for $X_{m,c}^{ss}$:

$$X_{m,c}^{ss} = \frac{R(p_s^{ss})^{\alpha_3}(p_m^{ss})^{\frac{1}{1-\alpha_m}} - (p_m^{ss})^{\alpha_1}(Q(p_s^{ss})^{\alpha_3} + S(p_s^{ss})^{\frac{1}{1-\alpha_s}}) + N(p_s^{ss})^{\alpha_2}(p_m^{ss})^{-1}}{\left(\alpha_m(1-\alpha_s)E(p_m^{ss})^{\alpha_1} - \alpha_s(1-\alpha_m)D(p_s^{ss})^{\frac{1}{1-\alpha_s}}(p_m^{ss})^{\alpha_3} - (\alpha_m - \alpha_s)A(p_s^{ss})^{\alpha_3} \right)} \tag{54}$$

where $R=\left(\frac{\alpha_m}{r^{ss}}\right)^{\frac{\alpha_m}{1-\alpha_m}}(\alpha_s-\alpha_m\alpha_s)A+T\frac{\alpha_{res}}{(\alpha_{c-}\alpha_{res})}$, $Q=\frac{\alpha_{res}}{(\alpha_{c-}\alpha_{res})}(G+U)$, $S=\left(\frac{\alpha_m}{r^{ss}}\right)^{\frac{\alpha_m}{1-\alpha_m}}(\alpha_s-\alpha_m\alpha_s)D$, $N=\frac{\alpha_{res}}{(\alpha_{c-}\alpha_{res})}M$

Finally $X_{s,c}^{ss}$ is given by:

$$X_{s,c}^{ss}=\frac{(1-\alpha_c)p_m^{ss}}{\alpha_c p_s^{ss}}\frac{R(p_s^{ss})^{\alpha_3}(p_m^{ss})^{\frac{1}{1-\alpha_m}}-(p_m^{ss})^{\alpha_1}(Q(p_s^{ss})^{\alpha_3}+S(p_s^{ss})^{\frac{1}{1-\alpha_s}})+N(p_s^{ss})^{\alpha_2}(p_m^{ss})^{-1}}{\left(\alpha_m(1-\alpha_s)E(p_m^{ss})^{\alpha_1}-\alpha_s(1-\alpha_m)D(p_s^{ss})^{\frac{1}{1-\alpha_s}}(p_m^{ss})^{\alpha_3}-(\alpha_m-\alpha_s)A(p_s^{ss})^{\alpha_3}\right)} \tag{55}$$

Substituting expressions for $X_{m,res}^{ss}$, $X_{s,res}^{ss}$ $X_{m,c}^{ss}$ and $X_{s,c}^{ss}$ back into (48) and (50), equating right hand sides of the resulting equations and expressing p_s^{ss}in terms of p_m^{ss} yields:

$$p_s^{ss}=\left(\frac{\alpha_{res}^{1-\alpha_{res}}\alpha_c^{\alpha_c}(1-\alpha_c)^{1-\alpha_c}}{p_{res}^{ss}\alpha_{res}^{\alpha_{res}}(1-\alpha_{res})^{1-\alpha_{res}}}\right)^{\frac{1}{\alpha_{res}-\alpha_c}}p_m^{ss}$$

Finally substituting the last equation into condition (51) and solving equation yields expression for p_m^{ss} in terms of p_{res}^{ss} after which p_s^{ss} is also derived as a function of p_{res}^{s} :

$$p_m^{ss}=(p_{res}^{ss})^{\frac{1-\alpha_c}{\alpha_{res}-\alpha_c}}I \tag{56}$$

$$p_s^{ss}=(p_{res}^{ss})^{\frac{-\alpha_c}{\alpha_{res}-\alpha_c}}J \tag{57}$$

where $I=\alpha_c^{\frac{\alpha_c(\alpha_{res}-1)}{\alpha_{res}-\alpha_c}}(1-\alpha_c)^{\frac{(1-\alpha_c)(\alpha_{res}-1)}{\alpha_{res}-\alpha_c}}(1-\alpha_{res})^{\frac{(1-\alpha_{res})-(1-\alpha_c)\alpha_{res}}{\alpha_{res}-\alpha_c}}\alpha_{res}^{\frac{\alpha_{res}(1-\alpha_c)}{\alpha_{res}-\alpha_c}}$, $J=\alpha_c^{\frac{\alpha_c\alpha_{res}}{\alpha_{res}-\alpha_c}}(1-\alpha_c)^{\frac{(1-\alpha_c)\alpha_{res}}{\alpha_{res}-\alpha_c}}\alpha_{res}^{-\frac{\alpha_{res}\alpha_c}{\alpha_{res}-\alpha_c}}(1-\alpha_{res})^{\frac{(1-\alpha_c)\alpha_{res}}{\alpha_{res}-\alpha_c}}$.

Now let's derive the profit maximizing input in steady state for housing construction firms. The profit maximazing conditions are given by:

$$q^{ss}\varepsilon(X_{res}^{ss})^{\varepsilon-1}(\overline{L_c}\ +g\overline{L_{uc}})^{1-\varepsilon}-n\varepsilon(X_{res}^{ss})^{\varepsilon-1}(\overline{L_c}\ +g\overline{L_{uc}})^{1-\varepsilon}=p_{res}^{ss} \tag{58}$$

$$q^{ss}(1-\varepsilon)(X^{ss}_{res})^{\varepsilon}(\overline{L_c}\ +g\overline{L_{uc}})^{-\varepsilon}-n(1-\varepsilon)(X^{ss}_{res})^{\varepsilon}(\overline{L_c}\ +g\overline{L_{uc}})^{-\varepsilon}=p^{ss}_l \tag{59}$$

Solving (58) for X^{ss}_{res} yields :

$$X^{ss}_{res}=\left(\frac{\varepsilon(q^{ss}-n)}{p^{ss}_{res}}\right)^{\frac{1}{1-\varepsilon}}(\overline{L_c}\ +g\overline{L_{uc}}) \tag{60}$$

This implies that quantity of new housing units produced in steady state is given by :

$$F^{ss}=\left(\frac{\varepsilon(q^{ss}-n)}{p^{ss}_{res}}\right)^{\frac{\varepsilon}{1-\varepsilon}}(\overline{L_c}\ +g\overline{L_{uc}}) \tag{61}$$

Using residential investment good market clearing condition I can write:

$$\left(\frac{\varepsilon(q^{ss}-n)}{p^{ss}_{res}}\right)^{\frac{1}{1-\varepsilon}}(\overline{L_c}\ +g\overline{L_{uc}})=(X^{ss}_{m,res})^{\alpha_{res}}(X^{ss}_{s,res})^{1-\alpha_{res}} \tag{62}$$

Finally housing market clearing condition and composite good market clearing condition in the steady state can be written as:

$$\delta_h h^{ss}_c+\delta_h g h^{ss}_{uc}=\left(\frac{\varepsilon(q^{ss}-n)}{p^{ss}_{res}}\right)^{\frac{\varepsilon}{1-\varepsilon}}(\overline{L_c}\ +g\overline{L_{uc}}) \tag{63}$$

$$c^{ss}_c+gc^{ss}_{uc}=(X^{ss}_{m,c})^{\alpha_c}(X^{ss}_{s,c})^{1-\alpha_c} \tag{64}$$

Equations (24)-(28) , (33)-(36), (42), (45)-(47),(52)-(57), (59)-(60), (62)-(64) together with conditions $K^{ss}_m=\psi k^{ss}$, $K^{ss}_s=(1-\psi)k^{ss}$ $N^{ss}_m=\phi(1+g)$ and $N^{ss}_s=(1-\phi)(1+g)$ represent system of equations which fully determines the steady state.

4 Loglinearization

Loglinearizing around steady state the optimilality conditions for households and firms and laws of motion of stochastic states yields the following equations:

$$\widetilde{c_{c,t}} = \frac{w^{ss}}{c_c^{ss}}\widetilde{w}_t + \frac{\overline{L_c} p_l^{ss}}{c_c^{ss}}\widetilde{p_{l,t}} + \frac{i^{ss} b_c^{ss}}{c_c^{ss}}\widetilde{i}_t + \frac{(i^{ss}+p_b^{ss})b_c^{ss}}{c_c^{ss}}\widetilde{b_{c,t}} - \frac{p_b^{ss} b_c^{ss}}{c_c^{ss}}\widetilde{b_{c,t+1}} - \frac{q^{ss} h_c^{ss}}{c_c^{ss}}\widetilde{h_{c,t+1}} + \frac{q^{ss} h_c^{ss}(1-\delta_h)}{c_c^{ss}}\widetilde{h_{c,t}} -$$

$$- \frac{q^{ss} h_c^{ss} \delta_h}{c_c^{ss}}\widetilde{q}_t$$

$$\widetilde{c_{uc,t}} = \frac{w^{ss}}{c_{uc}^{ss}}\widetilde{w}_t + \frac{\overline{L_c} p_l^{ss}}{c_{uc}^{ss}}\widetilde{p_{l,t}} + \frac{i^{ss} b_{uc}^{ss}}{c_{uc}^{ss}}\widetilde{i}_t + \frac{(i^{ss}+p_b^{ss})b_{uc}^{ss}}{c_{uc}^{ss}}\widetilde{b_{uc,t}} - \frac{p_b^{ss} b_{uc}^{ss}}{c_{uc}^{ss}}\widetilde{b_{uc,t+1}} - \frac{q^{ss} h_{uc}^{ss}}{c_{uc}^{ss}}\widetilde{h_{uc,t+1}} + \frac{q^{ss} h_{uc}^{ss}(1-\delta_h)}{c_{uc}^{ss}}\widetilde{h_{uc,t}} -$$

$$- \frac{q^{ss} h_{uc}^{ss} \delta_h}{c_{uc}^{ss}}\widetilde{q}_t - \frac{k^{ss}}{c_{uc}^{ss}}\widetilde{k_{t+1}} + \frac{(1-\delta_k+r^{ss})k^{ss}}{c_{uc}^{ss}}\widetilde{k}_t + r^{ss} k^{ss} \widetilde{r}_t$$

$$\widetilde{b_{c,t+1}} = \widetilde{q}_t + \widetilde{h_{c,t+1}}$$

$$\widetilde{\upsilon}_t = \frac{p_b^{ss}}{\upsilon^{ss}(c_c^{ss})^\gamma}\widetilde{p_{b,t}} - \frac{\beta^c i^{ss}}{\upsilon^{ss}(c_c^{ss})^\gamma}\widetilde{i_{t+1}} \quad - \frac{\beta^c p_b^{ss}}{\upsilon^{ss}(c_c^{ss})^\gamma}\widetilde{p_{b,t+1}} + \left(\frac{p_b^{ss}\gamma}{\upsilon^{ss}(c_c^{ss})^\gamma} - \gamma\right)\widetilde{c_{c,t+1}} - \left(\frac{\beta^c p_b^{ss}\gamma}{\upsilon^{ss}(c_c^{ss})^\gamma} + \frac{\beta^c i^{ss}\gamma}{\upsilon^{ss}(c_c^{ss})^\gamma} + \gamma\right)\widetilde{c_{c,t}}$$

$$\widetilde{h_{c,t+1}} = -\widetilde{c_{c,t+1}} + \left(\frac{\beta^c\theta}{q^{ss}h_c^{ss}} + \frac{\beta^c(1-\delta_h)}{h_c^{ss}} + \frac{mv^{ss}}{h_c^{ss}}\right)\widetilde{c_{c,t}} + \left(\frac{mv^{ss}}{\gamma h_c^{ss}} - \frac{1}{\gamma}\right)\widetilde{q}_t + \frac{mv^{ss}}{\gamma h_c^{ss}}\widetilde{\upsilon}_t + \frac{\beta^c(1-\delta_h)}{\gamma h_c^{ss}}\widetilde{q_{t+1}}$$

$$\widetilde{h_{uc,t+1}} = -\widetilde{c_{uc,t+1}} + \left(\frac{\beta^c\theta}{q^{ss}h_{uc}^{ss}} + \frac{\beta^c(1-\delta_h)}{h_c^{ss}}\right)\widetilde{c_{uc,t}} - \frac{1}{\gamma}\widetilde{q}_t + \frac{\beta^c(1-\delta_h)}{\gamma h_c^{ss}}\widetilde{q_{t+1}}$$

$$\widetilde{p_{b,t}} = \gamma\left(\widetilde{c_{uc,t}} - \widetilde{c_{uc,t+1}}\right) + \beta^{uc}\frac{i^{ss}}{p_b^{ss}}\widetilde{i_{t+1}} + \beta^{uc}\widetilde{p_{b,t+1}}$$

$$0 = \gamma\left(\widetilde{c_{uc,t}} - \widetilde{c_{uc,t+1}}\right) + \beta^{uc} r^{ss}\widetilde{r_{t+1}}$$

$$\widetilde{p_{s,t}} + \widetilde{z}_t + (\alpha_s - 1)\widetilde{K_{s,t}} + (1-\alpha_s)\widetilde{N_{s,t}} = \widetilde{r}_t$$

$$\widetilde{p_{m,t}} + \widetilde{z}_t + (\alpha_m - 1)\widetilde{K_{m,t}} + (1-\alpha_m)\widetilde{N_{m,t}} = \widetilde{r}_t$$

$$\widetilde{p_{s,t}} + \widetilde{z}_t + \alpha_s\widetilde{K_{s,t}} - \alpha_s\widetilde{N_{s,t}} = \widetilde{w}_t$$

$$\widetilde{p_{m,t}} + \widetilde{z}_t + \alpha_m\widetilde{K_{m,t}} - \alpha_m\widetilde{N_{m,t}} = \widetilde{w}_t$$

$$\widetilde{p_{res,t}} + (\alpha_{res} - 1)\widetilde{X_{m,res,t}} + (1-\alpha_{res})\widetilde{X_{s,res,t}} = \widetilde{p_{m,t}}$$

$$\widetilde{p_{res,t}} + \alpha_{res}\widetilde{X_{m,res,t}} - \alpha_{res})\widetilde{X_{s,res,t}} = \widetilde{p_{s,t}}$$

$$(\alpha_c - 1)\widetilde{X_{m,c,t}} + (1-\alpha_c)\widetilde{X_{s,c,t}} = \widetilde{p_{m,t}}$$

$$\alpha_c\widetilde{X_{m,c,t}} - \alpha_c\widetilde{X_{s,c,t}} = \widetilde{p_{s,t}}$$

$$p_{res}^{ss}\widetilde{p_{res,t}} = q^{ss}\varepsilon(X_{res}^{ss})^{\varepsilon-1}(La^{ss})^{1-\varepsilon}\widetilde{q}_t + (q^{ss}\varepsilon(X_{res}^{ss})^{\varepsilon-1}(La^{ss})^{1-\varepsilon} - n\varepsilon(X_{res}^{ss})^{\varepsilon-1}(La^{ss})^{1-\varepsilon})((\varepsilon-$$

$1)\widetilde{X_{res,t}} + (1-\varepsilon)\widetilde{La_t})$

$$p_l^{ss}\widetilde{p_{res,t}} = q^{ss}(1-\varepsilon)(X_{res}^{ss})^{\varepsilon}(La^{ss})^{-\varepsilon}\widetilde{q_t} + (q^{ss}(1-\varepsilon)(X_{res}^{ss})^{\varepsilon}(La^{ss})^{-\varepsilon} - n(1-\varepsilon)(X_{res}^{ss})^{\varepsilon}(La^{ss})^{-\varepsilon})(\varepsilon\widetilde{X_{res,t}} - \varepsilon\widetilde{La_t})$$

$$h_c^{ss}(\widetilde{h_{c,t+1}} - (1-\delta)\widetilde{h_{c,t}}) + gh_{uc}^{ss}(\widetilde{h_{uc,t+1}} - (1-\delta)\widetilde{h_{uc,t}}) = (X_{res}^{ss})^{\varepsilon}(La^{ss})^{1-\varepsilon}(\varepsilon\widetilde{X_{res,t}} + (1-\varepsilon)\widetilde{La_t})$$

$$c_c^{ss}\widetilde{c_{c,t}} + gc_{uc}^{ss}\widetilde{cu_{c,t}} = (X_{m,c}^{ss})^{\alpha_c}(X_{s,c}^{ss})^{1-\alpha_c}(\alpha_c\widetilde{X_{m,c,t}} + (1-\alpha_c)\widetilde{X_{s,c,t}})$$

$$K_m^{ss}\widetilde{K_{m,t}} + K_s^{ss}\widetilde{K_{s,t}} = gk^{ss}\widetilde{k_{t+1}}$$

$$\widetilde{b_{c,t+1}} = \widetilde{b_{uc,t+1}}$$

$$\widetilde{X_{res,t}} = \alpha_{res}\widetilde{X_{m,res,t}} + (1-\alpha_{res})\widetilde{X_{s,res,t}}$$

$$\widetilde{N_{m,t}} + \widetilde{N_{s,t}} = 0$$

$$\widetilde{La_t} = 0$$

$$X_{m,c}^{ss}\widetilde{X_{m,c,t}} + X_{m,res}^{ss}\widetilde{X_{m,res,t}} = (K_m^{ss})^{\alpha_m}(N_m^{ss})^{1-\alpha_m}(\alpha_m K_{m,t} + (1-\alpha_m)N_{m,t})$$

$$X_{s,c}^{ss}\widetilde{X_{s,c,t}} + X_{s,res}^{ss}\widetilde{X_{s,res,t}} = (K_s^{ss})^{\alpha_s}(N_s^{ss})^{1-\alpha_s}(\alpha_s K_{s,t} + (1-\alpha_s)N_{s,t})$$

$$\widetilde{z_t} = (1-a)\widetilde{z_{t-1}} + \widetilde{\xi_t}$$

$$\widetilde{\eta_t} = \rho\widetilde{\eta_{t-1}} + \widetilde{\zeta_t}$$

5 Calibration

Based on Davies and Heathcote (2003) α_s is set equal to 0.3 , α_m is set equal to 0.5 , α_c is set equal to 0.4 and α_{res} is set equal to 0.7 and ε is set equal to 0.5 . Following Kiyotaki and Moore the proportion of unconstrained households to constrained households g is set to 1.5 . Discount factor for unconstrained households β^{uc} is set equal to conventional 0.99 while the discount factor for credit constrained housheholds β^c is set at 0.97 , reflecting the fact that they are more impatient. Following Campbell and Cocco (2004) I set $\theta = 1.3$ and $\gamma = 2$. Based on construction cost index process calibration $\rho = 0.6$ and $\sigma = 1.5$. The first value of m in the Markov swithing process is set equal to 0.8 reflecting the initial situation with lower loan-to-value ratio, the second value is set to 0.95 refelcting the rapid liberalization of mortgage conditions which happened in the beginning of 2000s and the third value is set to 0.85 refelcting the consequent crisis in the mortgage market and tightening up of the mortgage conditions. Housing depreciation rate δ_h is set to 0.03 and capital depreciation rate δ_k is set to conventional 0.1 .

6 Results: Changes in transition and steady state comparisons

Let's start with considering the situation when there is one percent positive productivity shock in the intermediate good production sector. The positive productivity shock of 1% within present model results in increase in equilibrium wage by 0.56 % during the first period.

This increases the disposable income of households and generates increase in composite good consumption by 0.19% and by 0.245 % in housing consumption during the first four periods (these are the changes in total composite good consumption and housing consumption of both credit constrained and unconstrained households which are aggregated using calibrated relative share of unconstrained households in the economy). Increase in housing consumption leads to an increase in equilibrium housing price by 0.17 % which in turn brings to addtional increase in consumption during consequent periods by 0.13 % and increase in borrowing by credit constrained households by 0.1% The economy reaches the new steady state during 20 periods . In the new steady state the composite good consumption is higher by 0.32 % and housing consumption is higher 0.43 % , and the lifetime expected utility of the households is higher by 0.47 % . During the transition , due to positive dynamics of housing consumption and composite good consumption lifetime utility also rises by 0.12 %

Now let's consider what happens with the economy when there is a 1% increase of production costs in the sector producing residential investment good (increase in η_t) . This increase shifts down the production of residential investment good by 0.24% . As immediate reaction after the first period this also decreases demand for the intermediate goods and drives down the equilibrium wage by 0.15 % .This shifts down both composite good consumption and housing consumption. Decrease in the residential investment good production implies decrease in new housing production which decreases by 0.17 % during the first three periods. As a result price of housing increases by 0. 135 % . This leads to decrease in housing consumption by 0.09 % . However the composite good consumption which is positively related to housing price due to housing equity effect and opportunity for additional borrowing increases by 0.13 % after second period . Rise in consumption good demand promotes realocation of intermediate good supply to composite good production sector and increases again demand for intermediate goods and thus equilibrium wages . The economy converges to new steady state in 25 periods In the new steady state the model economy ends up with lower housing consumption by 0.17 % but higher composite good production by 0.20 % . The lifetime utility during transition decreases by 0.115 % but in the new steady state the total lifetime utility of the households is higher by 0.05 % .

Finally let's consider the implications of shift in the loan-to-value ratio from 0.8 to 0.95 , which reflects the rapid liberalization of mortgage market in the beginning of 2000s. The increase in the loan-to-value ratio leads to an increase in housing price which increases the value of outstanding housing equity and allows households to increase both housing consumption and composite good consumption . After the first four periods composite good consumption rises by 18 % and housing consumption rises by 20 % . They continue to increase during the further periods of transition which lasts 15 periods. In the new steady state the composite good consumption is higher by 26 % and housing consumption is higher by 29 % . The lifetime utility in the new steady state is higher by 45 % .

The situations analyzed above were characteristic for housing boom years which continued up to 2006 where majority of shocks for the housing market were positive . Now we can analyze what happened when suddenly the negative income shock and tightening of the mortgage markets affected the housing market. Suppose that we start in a steady with 1% higher productivity than the initial one and model economy experiences negative productivity shock which leads to negative income shock. However even if we consider again the negative productivity shock of 1 % the situation is not the same as going back to initial steady state . In the new steady state with higher productivity shock the constraints are again binding but the borrowing is higher and with lower income it is harder for the households to repay the debt . 1 % negative income shock leads to decrease in housing consumption by 0.36 % and decrease in composite good production by 0.42 % during the first 3 periods of transition and this decrease continues further for the composite good consumption good since lower housing demand leads to lower housing price and consumption is positively related to housing price . In the new steady state both housing consumption and composite good consumption are lower by respectively 0.43% and 0.51 % and lifetime utility is lower by 0.38 % . The situation is similar when economy shifts from state with loan-to-value ratio of 95 % to state with LTV of 85 %. Tightening of the mortgage conditions decreases demand for housing and decreases housing price . The value of outstanding housing equity decreases which makes it more difficult for credit constrained households to return debt and forces them to significantly cut consumption . After the first five periods composite good consumption decreases by 23 % and then continues to slightly decrease untill reaching the new staedy state. Housing consumption decreases by 25.5 % during the first 3 periods due to tightening of credit market and after third period starts to increase slightly due to lower housing price . Transition lasts 18 periods. In the new steady state the composite good consumption is lower by 28.2 % and housing consumption is lower by 23.5 % . The lifetime utility in the new steady state is lower by 48.6 % .