Irrational belief and credit spreads puzzle☆

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Abstract

In this paper,we extend the Merton's structured approach to incomplete market economy, in which investors may have irrational belief about the corporate's future cash flow. Our theoretical model implies irrational belief generates an additional risk .In detail, as distorted belief increases, the corporate values decreases, but the corporate value volatility and negative risk neutral skewness increase.Furthermore since risky corporate bonds are proportional to a short put on the corporate value, credit spreads widen .So irrational belief help to explain the credit spreads puzzle. Finally,we use empirical analysis to test our model results and find that the coefficient of distorted belief in the regressions for credit spreads, in control of many variables, still significant positive.

Key words:
Irrational belief, credit spreads puzzle, structural approach

1. Introduction

Merton (1974) structural approach essentially reveals the default of the corporate, which assumes that a corporate defaults on all its liabilities when the value of its assets at maturity is below a threshold. The approach has

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been useful in understanding the risky corporate debt as a proportion to a short put on corporate value, which has directly led to a recent revolution in measuring and managing credit risk similarly with option using the information on asset values and their volatilities.

However, recent empirical testing has cast doubts on the ability of the entire structural form family of models to simultaneously generate both default probabilities and credit spreads that match historical experience. Delianedis and Geske (2001) argue that measures of expected default losses based on structural models are too low to be consistent with the high observed credit spreads. With related calibration methodologies, Elton, Gruber, Agrawal, and Mann (2001)and Amato and Remolona (2003)find a similar conclusion. Extending this logic, Huang and Huang (2003) calibrate various structural form models to match the average solvency ratios of corporates of different rating categories, and in addition choose free parameters to match the historical default probabilities and recovery rates on defaulted bonds, and find that the model-generated spread accounts for less than a third of the total spread over treasuries on investment-grade corporate bonds. By constraining the model to match historical default losses, the calibration results of these authors is seemingly robust to the specification of the asset value process or the default threshold since changing either assumption leads to roughly equivalent variation in expected default losses and credit spreads. This problem has been called the credit spreads puzzle. Additionally, Chen, Collin-Dufresne, and Goldstein (2009) have contributed to a deepening of the puzzle. These authors argue that the credit spreads and equity premium puzzles are related since they measure risk premiums of different liabilities on the same underlying asset value process. It is therefore natural to consider models in the literature that have been successful in generating a large equity premium and assess their ability in addressing the credit spreads puzzle.

After reviewing the outstanding articles on credit spread puzzle,one paper Stephen G. Cecchetti, Pok-sang Lam and Nelson C. Mark (2000)causes my interest, in which authors doubt fully rational agents supposition in standard Robert E. Lucas, Jr. (1978) representative agent asset-pricing model of an endowment economy. By an alternative approach that allows for small departures from rationality in an otherwise, they success in explanation the equity premium puzzle. Following this, a lot of articles such as D Hirshleifer (2001), Z Chen, L Epstein (2002), LP Hansen (2007) do referent research and find irrational or distorted belief can help explain some rational paradigm. In broad term, it argues that some financial phenomena can be better understood using models in which some agents are not fully rational .A survey of behavioral finance Nicholas Barberis Richard Thaler(2002) mention "a series of theoretical papers showing that in an economy where rational and irrational traders interact, irrationality can have a substantial and long lived impact on prices". Then in Kogan et al. (2006), they prove that irrational can survival for long term and give the long equilibrium results of rational and irrational trader under different conditions. Considering the equity premium and credit products are written on the same corporate asset, so it's natural to consider if irrational belief factors affect the credit spread. With this question, we begin our research.

Our model follows Kogan et al. (2006) the irrational assumptions and structural approach about credit risk framework , discusses credit risk model with irrational belief and its impacts. We show in equilibrium, irrational beliefs force investors to take speculative positions against each other and therefore generate endogenous relative wealth fluctuation and impact on pricing, which even give an explanation to credit spread puzzle. Therefore, irrational belief as a risk factor, which ignored by classical credit risk has important impact on practice and is discussed in this paper.

When we regress our model with identical belief, it has consistent result with the classical Merton (1974) model. In other words, Merton model is specific model under fully rational belief.

The paper is organized as follows.Section III introduces our structural equilibrium model with irrational beliefs. Then section IV presents and discusses the equilibrium solutions for risky or defaultable corporate bond prices, equity and credit derivatives. Section V analyzes the impact of irrational traders on prices volatility and credit spreads. Section VI empirical analysis . Section V is the conclusion. Proofs are collected in the appendix.

2. Literature review

2.1. Credit risk literature review

For modeling credit risk, two classes of models mainly exist: structural and reduced form. Structural models originated with Merton (1974), which specifies the firm-value process and assumes that default is triggered on the maturity date if firm value is less than the value promised to the bondholders. Thus, the value of defaultable bond can be considered as an option of value of the firm and be measured by option pricing model such as Black, and Scholes (1973) . The advantage of this model reveals the endogenous relationship between default and the value of the firm. Meanwhile it finds a methold to solve the credit risk. Then, Black and Cox (1976) develop the first passage approach (FPA), which modifies the definition of the default to first time (stop time) when firm value is less than the bond value. After these two basic models, many researchers expanded structural model from different views, such as (1) stochastic rate process: Kim et al. (1993), Longstaff and Schwartz (1995) ; (2) more rational default barrier: Giesecke and Goldberg (2004) define time-varying default barrier.François and Morellec (2004) and Kay (2004)point out bankruptcy codes often grant firms an extended period of time to reorganize operations after a default, when the restructuring is not successful and son on .

The other important model of credit risk is reduced form model, which is originated with Jarrow and Turnbull (1992), and subsequently studied by Jarrow and Turnbull (1995), Duffie and Singleton (1999) among others. Reduced form model, by exogenous hazard rate, can reflect default of the firm at sudden date. The main improvement of this model focuses on the hazard rate, such as Li (2000) separates the influent factors of hazard rate into common factors (macroeconomic factor , industry factor) and idiosyncratic factors. Common factors are measured by copula ,which grasps the default correlation among different firms. Duffie and Pan (2001) and Duffie et al. (2003) introduce the unexpected jump risk into the hazard rate, which can model the arrival of news in the economy. Above all, reduced form model can reflect the sudden default of the firm rather than the endogenous default reason. Though the reduced form model is easy to model expansion, it cannot provide the instinct explanation to default of firm.

In this paper, to reflect the endogenous default reason and investor behavior,we choose the structural credit risk model framework.

2.2. investor's belief literature review

For decades, an obvious schism has divided the field of finance: The success of the efficient market hypothesis (EMH) in explaining prices that always reflect all the information of underlying intrinsic values has clashed with the inefficient and irrational phenomenon, especially financial crisis. Only since the mid-1980s has there been a serious attempt to explore the possibility that inefficient markets are not always as orderly as might be suggested by the efficient market advocates. For example, as the "noise trader" theories of Kyle (1985) and Black (1986) suggest, if some investors trade on a "noisy" signal that is unrelated to fundamentals, then asset prices will deviate from their intrinsic value. DeLong et al (1990a,b)use a partial equilibrium model and suggest that traders with wrong beliefs may survive in the long-run since they may hold a portfolio with excessive risk but also higher expected return and therefore their wealth can eventually outgrow that of rational traders which markes a new epoch in behavioral finance. Since that, many studies have been devoted to analyze this issue, e.g. Basak (2000) studies the exogenous risk which is uncorrelated with fundamentals and points out that distorted beliefs as the risk factor affects equilibrium prices .Xiong and Scheinkman (2003) use overconfidence to generate a parameterized model of different beliefs and explain US Internet stocks during the period of 1998-2000. Dumas et al (2009) study the relation between beliefs distortion, deriving from overconfidence of some agents in the economy, and the excess volatility of stock returns. In recent papers, Xiong and Yan (2009) model investors' beliefs by supposing that the parameters of the learning models of different investors are different. Buraschi and Alexei (2006) consider investors who have different belief on the dividend growth rate and information signal may have different consumption plans, optimal portfolios and asset prices. Until now, Basak (2000), Dumas et al (2009), Jouini and Napp (2005), Buraschi and Alexei (2006), David (2007), and Li (2007) provide equilibrium models to study the effects of investor's distorted beliefs on a variety of issues, including asset price volatility, interest rates, equity premium, and the option implied volatility.

3. The model

3.1. Assumptions

Assumption 3.1. The economy evolves in continue time. Uncertainty is described by a one-dimensional standard Brownian motion B_t for $0 \leq t < T$. Defined a probability space (Ω, F, P) , where$F$ is the augmented filtration generated by standard Brownian motion B_t, P is the objective (physical) probability measure.

Assumption 3.2. Under probability measure$P$, the expected cash flows of the firm A_t, can be described by a diffusion stochastic process:

$$dA_t = \mu A_t dt + \sigma A_t dB_t \tag{1}$$

Where μ and σ are respectively instantaneous expected rate of return on the firm per unit time and volatility of the return on the firm per unit time and B_t is a standard Brownian motion.

Assumption 3.3. There are two kinds of investors: rational belief investors and irrational or distorted belief investors, where rational belief investors has consistent belief with objective probability of the states of the economy, however, irrational or distorted belief traders, following the definition with Stephen G. Cecchetti, Pok-sang Lam and Nelson C. Mark (2000), are the respective agent whose beliefs about endowment growth are distorted

Under the subjective probability measure Q, assume the firm's expected cash flow follow the diffusion process:

$$dA_t = A_t(\mu + \sigma^2\eta)dt + \sigma dB_t^Q \tag{2}$$

$$dB_t = (\sigma\eta)dt + dB_t^Q \tag{3}$$

Where B_t^Q is standard Brownian motion under the measure Q and η is a constant, measuring the irrational degree. When$\eta > 0$, irrational investor is optimistic about the future of the firm and over estimates expected growth rate of the firm; otherwise, when $\eta < 0$, irrational investors is pessimistic about the future of the firm and underestimate the expected growth rate of the firm.

As η is a constant, the probability measure of irrational investor Q is absolutely constituted with the true probability measure P. Two kinds of investors have identical belief in zero probability such that $\xi = (dQ/dP)_t$ is the density of probability measure Q to P(Randon-Nikodym derivative). According to the Girsanov theorem, we can get following :

$$\xi_t = e^{-\frac{1}{2}\eta^2\sigma^2 t + \eta\sigma B_t} \tag{4}$$

Assumption 3.4. Both traders have logarithmic preference [1]at time T :

$$U(C) = log(C_T) \tag{5}$$

The rational trader follows the objective probability measure and optimizes his expected utility

$$E^P(logC_{r,T}) \tag{6}$$

The irrational investor optimizes expected utility under subjective belief Q, which can be transferred to probability measure P by the density ξ_t,

$$E^Q(logC_{n,T}) = E^P(\xi_t logC_{n,T}) \tag{7}$$

[1]logarithmic preference is popular in recent papers, such as Xiong and Yan (2009)and so on.

3.2. The Equilibrium

Definition 3.1. *Equilibrium: An equilibrium consists of a unique allocation between the representative agents that such that (1) given equilibrium prices, all agents in the economy solve the optimization expected utility subject to their budget constraint. (2) Good and financial markets clear.*

The Equilibrium of the economy defined above shows complete nature in the market. Sophisticated position against each other is always present as long as the volatility of firm's return remains non-zero almost surely. Consequently, the equilibrium allocation is efficient and can be characterized as the solution to a central planner's problem.

$$Max(logC_{r,t} + \xi_t logC_{n,t}) \tag{8}$$
$$\text{s.t.} C_{r,T} + C_{n,T} = A_T$$

Above dynamic optimal control problem can be transformed into a static one and solved using the martingale approach noted in Cox and Huang (1989), Cuoco and He (1994), Karatzas and Shreve (1998) et al.

Proposition 3.1. *In equilibrium, the allocation between the two agents is :*

$$C_{r,T} = \frac{A_T}{1+\xi_T} \tag{9}$$

$$C_{n,T} = \frac{\xi_T A_T}{1+\xi_T}$$

The equilibrium state price density at time t is given by $\phi = \frac{(1+\xi_T)A^{-1}}{E_t[(1+\xi_T)A_T^{-1}]}$. The price of a financial security with the terminal payoff Z_T is given by:

$$P_T = \frac{E_t[(1+\xi_T)A_T^{-1}Z_T]}{E_t[(1+\xi_T)A_T^{-1}]} \tag{10}$$

Application of the equilibrium state price density ϕ gives the equilibrium value of the firm after the competition of two representative agents:

$$V_T = \frac{1+\xi_T}{E_t[(1+\xi_T)A_T^{-1}]} = \frac{1+\xi_T}{1+e^{-\sigma^2\eta(T-t)}\xi_t} \frac{1}{e^{(-\mu+\frac{1}{2}\sigma^2)T+\frac{1}{2}\sigma^2}(T-t) - \sigma B_t} \tag{11}$$

In the extreme cases when only the rational or the irrational agent is present, the values of the firm, denoted by V_t^* and V_t^{**} , respectively, are given by :

$$V_t^{**} = V_t^* e^{\eta\sigma^2(T-t)} \tag{12}$$
$$V_t^* = e^{\mu T+\sigma B_t-\sigma^2 T+\frac{1}{2}\sigma^2 t}$$

To state our result formally, we define the relative wealth shares of the rational and irrational agents[2]:

$$\alpha_t = \frac{W_{n,t}}{W_{r,t}} = \xi_t \tag{13}$$

Compared expressions (11) with (12) , V_t can be given the some weighted form of V_t^{**} and V_t^* :

$$V_t = \frac{1}{1+g} V_t^* + \frac{g}{1+g} V_t^{**} \tag{14}$$

The weighed expression of V_t shows the compete character between the two group agents with extreme firm's value V_t^{**} and V_t^*. The weight coefficient $g = \xi_t e^{\frac{1}{2}\sigma^2(\eta^2-2\eta)(T-t)}$ is related to relative wealth shares α_t .

Based on the value of firm, we consider the value of defaultable bond, equity and credit default swaps.

4. The value of securities under irrational traders

Following the ideas of Merton's structural credit risk model, the firm's capital structure is composed of equity and a zero coupon bond. The firm's equity is viewed as a European call option on the firm's assets with maturity T and a strike price equal to the face value of the debt K. Debt covenants grant bond investors absolute priority: if the firm cannot fulfill its payment obligation, then bondholders will immediately take over the firm. Hence the bondholder equivalently has the value of the firm meanwhile short a call option. The bondholder's contingent payoff is given by:

$$Z_T^B = V_T - Max(V_T - K, 0) = \begin{cases} V_T & \text{if } V_T < K \\ K & \text{if } V_T < K \end{cases} \tag{15}$$

[2] $\alpha_t = \frac{W_{n,t}}{W_{r,t}} = \frac{E_t^Q(logC_{n,T})]}{E_t^P(logC_{r,T})} = \xi_t$

Faced on the defaultable bond, People need new financial tools to hedge and transfer the default risk. At that moment, credit derivatives emerged. As the cornerstone product of the credit derivatives market, the credit default swap (CDs) since it emerged in 1997 and began to grow rapidly from 2003. By the end of 2007, the CDS market had a notional value of $45 trillion which represents about fifty percent of the credit derivatives market[3]. CDs are bilateral contracts between a buyer and seller like a insurance contract, under which are often used to manage the credit risk. Typically, credit default swap (CDS) is a contract where the buyer is entitled to payment from the seller of the CDS if there is a default by a particular firm.It means with the credit default protection of CDS contract, bondholder with the defaultable firm bond B_t equivalently owns the riskless bond value K

$$CDs_t + B_t = K \tag{16}$$

To simplify considering complex payment methods of CDs such as payment in kind, periodic payment, we assume that cash payment one time. Hence at time T, the contingent payoff of the CDs is given by:

$$Z_T^{CDs} = V_T - Max(K - V_T, 0) = \begin{cases} K - V_T & \text{if } V_T < K \\ 0 & \text{if } V_T < K \end{cases} \tag{17}$$

Proposition 4.1. *Under both rational and irrational traders existence, the equilibrium price of defaultable firm bond is given by:*

$$B_t = K\frac{\Phi(-d_1) + g\Phi(\sigma\eta\sqrt{T-t} - d_1)}{1+g} - V_t^*\frac{\Phi(-d_2) + ge^{\eta\sigma^2(T-t)}\Phi(d_2 - \sigma\eta\sqrt{T-t})}{1+g} \tag{18}$$

where,$d_1 = \frac{c+\sigma(T-t)}{\sqrt{T-t}}$, $d_2 = d_1 - \sigma\sqrt{T-t}$, c=$\frac{\ln K - \mu T}{\sigma} - B_t + \frac{1}{2}\sigma T$.

Corollary 4.1. *In the extreme case when only the rational is present, the equilibrium bond prices is*

$$B_t^* = V_t^*\Phi(d_2) + k\Phi(-d_1) \tag{19}$$

Compared with Merton (1974) , it has the same expression ignored the time values of money. so we can say that Merton (1974) is a particular case

[3]British Banker Association Credit Derivatives Report

under expression (19), in other words, expression (19) is an expansion of the result in Merton (1974) under heterogeneous belief.

Considering the equity structure of the firm, we assume that equity is composed of issuing stock and total amount one unit. Hence each equity or each stock share is given by:

$$Z_T^S = V_T - B_t = \begin{cases} 0 & \text{if } V_T < K \\ V_T - K & \text{if } V_T < K \end{cases} \tag{20}$$

Proposition 4.2. *Under both rational and irrational traders existence, the equilibrium price of credit default swaps $CDs_t$and each equity S_t are respectively given by*

$$CDs_t = K\frac{\Phi(d_1) + g\Phi(d_1 - \sigma\eta\sqrt{T-t})}{1+g} - V_t^* \frac{\Phi(d_2) + ge^{\eta\sigma^2(T-t)}\Phi(d_2 - \sigma\eta\sqrt{T-t})}{1+g} \tag{21}$$

$$S_t = V_t^* \frac{\Phi(-d_2) + ge^{\eta\sigma^2(T-t)}\Phi(\sigma\eta\sqrt{T-t} - d_2)}{1+g} - K\frac{\Phi(-d_1) + g\Phi(\sigma\eta\sqrt{T-t} - d_1)}{1+g} \tag{22}$$

In the extreme case when only the rational is present, the credit default swaps prices and each equity are respectively given by $CDs_t^* = K\Phi(d_1) - V_t^*\Phi(d_2)$,$S_t^* = V_t^*\Phi(-d_2) - k\Phi(-d_1)$; in contrast, when only irrational trader is present $CDs_t^{**} = K\Phi(d_1 - \sigma\eta\sqrt{T-t}) - V_t^{**}\Phi(d_2 - \sigma\eta\sqrt{T-t})$ and $V_t^{**} = K\Phi(\sigma\eta\sqrt{T-t} - d_2) - K_t^{**}\Phi(\sigma\eta\sqrt{T-t} - d_1)$

From the endogenous joint relationship showed in the expressions (18), (21) and (22), we note that irrational belief simultaneously affects bond, equity, and credit default swaps markets, no matter irrational traders whether participate all markets at the same time or not. The intuitive reason is they have the same underlying asset. The irrational belief in one market surely reflects the expectation of underlying asset, which also influences other derivatives of underlying asset. In other words, bubbles in one market may infect among different markets. That is why crisis always accompany several different market crunches.

5. Model Predications

5.1. Volatility amplification

Different beliefs cause the Agents' different speculative positions. Competition against different speculative positions leads to amplifying volatility.

Intuitively, firm asset prices are determined by agents' wealth-weighted average belief about future cash flows in equation (14). Since agents who are more optimistic about future rates bet on these assets rising against more pessimistic agents, any positive news about future rates would cause wealth to flow from pessimistic agents to optimistic agents, making the optimistic belief carry a greater weight. The relative-wealth fluctuation thus amplifies the impact of the initial news on asset yields. As a result, a higher belief dispersion increases the relative-wealth fluctuation and also increases the asset yield volatility and implied volatility of the asset derivatives. We summarize this intuition in the following proposition, and provide a formal proof in Appendix.

Proposition 5.1. *Volatility increases with the belief dispersion between the two groups of agents.*

PROOF. 5.1:

$$V_t = \frac{1+\xi_t}{1+e^{-\sigma^2\eta(T_t)}} \frac{1}{e^{(-\mu+\frac{1}{2}\sigma^2)T+\frac{1}{2}\sigma^2(T-t)-\sigma B_t}}$$

Application of Itôs Lemma, $Vol(dV_t)$ is:

$$\sigma_{V,t} = \sigma+\sigma\eta(1-\frac{\xi_t}{1+\xi_t})-\sigma\eta[1-\frac{\xi_t}{\xi_t+e^{-\eta\sigma^2(T-t)}}] = \sigma+\sigma\eta[\frac{\xi_t}{\xi_t+e^{-\eta\sigma(T_t)}}-\frac{\xi_t}{1+\xi_t}]$$

If the irrational coefficient $\eta \neq 0$, such as $\eta > 0$, which implies $e^{-\eta\sigma^2(T-t)} < 1$, thus $\frac{\xi_t}{\xi_t+e^{-\eta\sigma(T_t)}} - \frac{\xi_t}{1+\xi_t} > 0$, and hence $\sigma_{v,t} > \sigma$.

Similarly, when $\eta < 0$, it implies $e^{-\eta\sigma^2(T-t)} > 1$, thus $\frac{\xi_t}{\xi_t+e^{-\eta\sigma^2(T-t)}} - \frac{\xi_t}{1+\xi_t} < 0$, also hence $\sigma_{V,t} > \sigma$.

Above all, $\sigma_{V,t} > \sigma$ is always right as long as the inequality $\eta \neq 0$. Hence, the irrational belief amplifies the volatility of underlying asset and implied volatility of the derivatives. □

Excess volatility puzzle of asset markets (including bond and equity market) is always hot issue in finance field. The proposition of volatility amplification helps explain the puzzles. Shiller (1979) shows that the observed bond yield volatility exceeds the upper limits implied by the expectations hypothesis and the observed persistence in short rates. Gurkaynak et al. (2005) also document that bond yields exhibit excess sensitivity to particular shocks, such as macroeconomic announcements. Furthermore, Piazzesi and Martin (2006) find that by estimating a representative-agent asset pricing model with recursive utility preferences and exogenous consumption growth and inflation, the model predicts less volatility for long yields relative to short yields. Relating to this literature, Proposition5.1 shows that extending standard representative-agent models with different expectations can help account for the observed high asset prices yield volatility.

5.2. Firm Value Skewness and Credit Spread

As investor's distorted belief increases, the model generates a negative endogenous co-movement between the value of the firm and the firm value volatility, even if the local volatility of the firm cash flows is constant. This feature implies a moderate negative skewness of the physical distribution of the firm value, which generates a moderate increase of the physical probability of default. This positive relation between default probabilities and volatility is consistent with the recent evidence documented in Bharath and Shumway (2007).It is well-known that negative skewness can also be obtained in partial equilibrium models with jumps; see Pan (2002), in option pricing, and Zhang, Zhou, and Zhu (2006), Cremers, Driessen, and Maenhout (2007) and Tauchen and Zhou (2006), in credit risk. In our model, the negative skewness arises endogenously, even if cash flows and securities prices do not include a jump component, and follows from the form of the equilibrium stochastic discount factor in the economy with irrational beliefs.

To understand this point, from Proposition5.1 and Ito's Lemma, the diffusion term of the dynamics of each individual state price densities is given by:

$$d\phi_i(t)/\phi_i(t) - E(d\phi_i(t)/\phi_i(t)) = (s_i(t) - 1)\sigma\eta dB_t \tag{23}$$

Where$s_i(t) = \frac{c_i(t)}{A(t)}$is the equilibrium allocation of agent i to total A(t). It follows that the volatility of the individual state prices is asymmetric and systematically related to the allocation of the agents in the economy: A positive cash flow shock lowers the volatility of the individual stochastic discount factors, and vice versa. This feature is due to the decreasing marginal utility of consumption of the two investors and generates endogenously the negative skewness in the economy.It is consistent with recent empirical testing Bharath and Shumway(2007), which shows the negative skewness relationship even including jump risk in partial equilibrium model. This feature of negative skewness directly increases the default probability and hence widen credit spread.

6. Empirical Analysis

In this section, we test the main empirical predictions of our model. We analyze in a set of panel regressions the impact of irrational belief on corporate credit spreads. In our model, beliefs distortion unambiguously increases credit spreads and the option implied volatility. Therefore, we expect a positive sign for the coefficient of irrational belief in these regressions.

6.1. Data

In empirical analysis, we use four data sets for each firm to test the implication of the model on credit spread, including credit spread data, investor forecasts index data, macro-financial data and firm-specific data.

(1)CDS Spread .We collect monthly CDS spreads from a Bloomberg database. For our analysis, we use the composite spread of US dollar-denominated five-year CDS contracts written on both senior and subordinated debts of North American obligors. Although the maturity of the composite spread can range between six months and 30 years, five-year CDS contracts have become the most common in recent research. For example, Cao, Yu and Zhong (2010) estimate that more than 85 percent of all quotes in 1999 and 2006 are for five-year contracts. We eliminate obligors with missing observations between the start and last dates of their coverage. Combining all variables with other data sets, we arrive at a final sample of 104 firms during the period from Feb 2006 to Jan 2010.

(2)Investor irrational forecasts index data .To obtain a proxy of beliefs irrational forecast, we use analyst forecasts of earnings per share, from the Institutional Brokers Estimate System (I/B/E/S) database. This database contains individual analysts'forecasts organized by the date the forecasts was made and the last date the forecast was revised and confirmed as accurate. Diether, Malloy and Scherbina(2002) define belief dispersion as the standard deviation of earnings forecasts scaled by the absolute value of the mean earnings forecast. In our paper,we choose this variable as proxy of irrational belief.

(3)Options and stock Data .Options and stock data are taken from Bloomberg. The database may appear convenient to pull referent data by matching the specific firms' tickers. With option trade, we use option trading volume, implied volatility and 1st month implied volatility skewness. Stock returns and volume are dependent variables in the regression for credit spread.

(4) macro-financial data.Previous empirical literature has suggested some macro financial variables affect credit spread such as a risk-free benchmark yield, liquidity of corporate and treasury bonds and so on. To focus on the additional explanation of investor's irrational belief, we control several of these variables in our regression models. For the business cycle and term structure effects, we use the Standard and Poor's 500 return and non-farm payroll from Untied States Department of labor web. For systematic risk, we use the market excess return and two Fama and French factors HML and SML from the Kenneth French web .

(5) firm-specific data .We additionally control some firm-specific variables, including financial leverage, book value per share, and return on common equity from Bloomberg.

Table 1 Summary Statistics

For mainly variable, Panel A reports the cross-sectional summary statistics of the time-series means of104sample firms. Panel B reports the summary statistics of market variables.

Panel A: Firm-Level Variables

	Mean	Min	Max	Std. Dev.
Credit spread	287.69	3.00	4328.64	408.31
Implied volatility(%)	33.54	22.88	64.30	9.38
Implied volatility skew	-22.53	-51.40	-1.37	12.32
Leverage(%)	65.52	31.9	89.47	47.38
Firm Stock Return(%)	-1.3	-4.99	49.71 .	16.09
Book value per share	20.84	1.095	54.49	12.05
Irrational belief	53.46	0.10	3650.63	426.87

Panel B: Market-Level Variables

	Mean	Min	Max	Std. Dev.
Risk free rate(%)	0.303	0.02	0.44	0.013
Rm-Rf (%)	-0. 91	-19	5.00	0.47
HML (%)	0.064	-4.88	4.49	0.20
SMB(%)	0.017	-3.90	5.34	0.22
Non-farm payroll(/1000)	136.77	133.55	137.95	1.07
S&P 500 return (%)	-0.79	-17.09 .	4.82	14.41

6.2. *Empirical results*

In the regressions for credit spreads, we investigate the relevance of distorted belief with respect to several empirical models studied in the credit risk literature. We classify the control variables used in our credit spreads regressions according to the following list of models:

(1) Macro-financial variables,
(2) Fama-French factors and option implied-volatility,
(3) Full model without irrational belief proxy,
(4) Full model with irrational belief proxy.

Table 2 OLS Panel regression results from model (1) to model (4)

Model	（1）	（2）	（3）	（4）
Credit spread				
constant	354.82*** (6.64)	-183.09*** (-8.65)	-115.74*** (-5.09)	-101.59*** (-4.48)
Irrational belief Index	406.39*** (4.87)	201.07*** (3.04)		145.12*** (3.43)
Implied volatility		6.88*** (16.91)	6.77*** (22.19)	6.19*** (16.43)
Implied volatility Skew			0.26*** (3.20)	0.28*** (3.41)
Option volume			-0.11** (2.18)	-0.11** (-2.34)
Risk-free rate	-32.13*** (-8.17)	10.44*** (2.63)	9.33** (2.16)	6.64* (1.65)
S&P 500 returns	-298.69*** (-2.83)		-10.40 (-1.26)	-5.23 (-0.70)
Non-farm payroll（/1000）	-24.76*** (-6.30)		1.27 (0.37)	1.97 (0.53)
Stock returns	-58.12*** (-4.43)	-48.79*** (-7.31)	-33.95 *** (-6.98)	-44.49*** (-6.76)
Stock volume(/1000)			9．92*** (6.01)	8.99*** (5.48)
Leverage			0.16 (0.58)	0.13525 (0.49)
Book value per share			-1.61*** (-4.26)	-1.529*** (-4.31)
10 swap rate/100			20.45 (1.00)	0.16 (0.91)
Rm-Rf		52．63*** (5.05)	12.62 (1.56)	9.78 (1.34)
SMB		0.31* (1.90)	0.10 (0.45)	0.11 (0.59)
HML		0.11 (0.66)	0.23 (1.22)	0.14 (0.79)
Adjusted R2	0.48	0.63	0.65	0.73

*,** and ***denotes significance at the 10%,5% and 1%level respectively.
All estimate use autocorrelation and heteroscedasticity consistent t statistics.

In model (1), we research the impact of the irrational belief, after controlling the macroeconomic factors. CollinGoldstein and Martin (2001)found that stock return and risk-free rate have significant impact on the credit spread. Recently, Huang and kong(2007)found that macroeconomic factors, such as non-farm payroll have obvious explanation power on the credit spread, so in the model(1), we add them as controlling variables. After controlling these variables, we can see investor's irrational belief still have obvious impact on the credit spread. In model (2),we introduce the Fama-Frech factors as controlling factors, since Schaefer and Strebulaev (2008)suggest Fama-Frech factors have significant impact on the corporate bonds. The results in column 2 in table 2 show irrational belief is still highly significant and only the SMB factor additionally is .However,this last finding is not robust to the inclusion of further control variables in column 3. Compared model(3)with model(4)we focus on finding the explanation power of the irrational belief on credit spreads after including all the referent variables. In all, the impact of irrational belief on corporate credit spreads is significate and robust with respect to common control variables.

7. Conclusions

In this paper, we study the irrational belief on the credit spread theoretically and empirically. Firstly we extend the Merton's structured approach to incomplete market economy, in which investors may have distorted belief about the company's future. The features generate an additional risk, which help to explain the credit spreads puzzle. Next we are analyzed through an empirical research results of the theoretical model, the main conclusions are as follows:

Irrational beliefs widen corporate credit spreads. In our theoretical model, distorted belief generates trading patterns in which risk is transferred from the pessimistic to the optimistic investors. As distorted belief increases, the corporate values decreases, but the firm value volatility and negative risk neutral skewness increase. Since defaultable bonds are proportional to a short put on the firm value, credit spreads widen. Empirical analysis shows that the coefficient of distorted belief in the regressions for credit spreads, in control of many variables, still significantly.

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Appendix

PROOF. 3.1:

To solve for the equilibrium, we can use the martingale approach, originally developed by Cox and Huang (1989), in its extension to the case of heterogeneous beliefs; see, among others, Cuoco and He (1994), and Karatzas and Shreve (1998). The representative agent in the economy faces the following optimization problem:

$$Max(logC_{r,t} + \xi_t logC_{n,t}) \tag{24}$$

$$\text{s.t.} C_{r,T} + C_{n,T} = V_T$$

Optimality of individual consumption plans implies the following form

$$\xi_T = \frac{U_r'(C_{r,T})}{U_r'(C_{r,T})} = \frac{y_r \varphi_r}{y_n \varphi_n}$$

Where $U_r'(C_T) = \frac{1}{C_T}$ is the marginal utility function, which is assumed the same among agents. $y_i(i = r, n)$ is Lagrange multiplier and $\varphi_i(i = r, n)$ is the state price density under different subjective probability measure.

Under the probability measure of rational, the first order condition for rational agent is: $\frac{1}{C_{r,T}} = \xi_T \varphi_r$, and the first order condition for agent two is: $\xi_T \frac{1}{C_{n,T}} = \xi_T \varphi_r$. Market cleaning implies: $C_{n,T} + C_{r,T} = A_T$. By inserting in these formulas the first order condition, the optimal consumption policies of two agents are:

$$C_{r,T} = \frac{A_T}{1 + \xi_T} \qquad C_{n,T} = \frac{\xi_T A_T}{1 + \xi_T}$$

under P probability measure, the state price density is $\varphi_r = \frac{(1+\xi_T)A_T^{-1}}{E_t[(1+\xi_T)A_T^{-1}]}$, at time t , the prices of security with $Z_T$contingent payoff

$$P_T = \frac{E_t[(1 + \xi_T)A_T^{-1} Z_T]}{E_t[(1 + \xi_T)A_T^{-1}]}$$

Since the contingent cash flow of firm at time T is $Z_T^{V_t} = A_T$, the value of firm is $V_t = \frac{1+\xi_T}{E-t(1+\xi_T)A^{-1}T}$.

By inserting in these formulas the Properties of moment generating function, the denominator of the value of firm $E_t(1 + \xi_T)A_T^{-1} = E_t[A_T^{-1}] + E_t[\xi_T A_T^{-1}] =$

$e^{(\frac{1}{2}\sigma^2-\mu)T-\sigma B_t}+e^{\frac{1}{2}\sigma^2(T-t)}$. In the same way with the numerator, the market price of value of firm follows in closed form:

$$V_T=\frac{1+\xi_T}{E_t[(1+\xi_T)A_T^{-1}]}=\frac{1+\xi_T}{1+e^{-\sigma^2\eta(T-t)}\xi_t}\frac{1}{e^{(-\mu+\frac{1}{2}\sigma^2)T+\frac{1}{2}\sigma^2(T-t)-\sigma B_t}}$$

In the extreme situation, when only the rational investor or irrational investor is respective, it implies the optimal consumption is $C_{r,T}=A_T$ or$C_{n,T}=A_T$, and the extreme value of the firm is $V_t^*=e^{\mu T+\sigma B_t-\sigma^2T+\frac{1}{2}\sigma^2t}$ or $V_t^{**}=V_t^*e^{\eta\sigma^2(T-t)}$.

Compared with the extreme value of the firm, V_t is the weighted form of extreme situation

$$V_t=\frac{1}{1+g}V_t^*+\frac{g}{1+g}V_t^{**}$$

$g=\xi_te^{\frac{1}{2}\sigma^2(\eta^2-2\eta)(T-t)}$ is related to relative wealth shares α_t . □

Proofs 4.1 is similar with that of 4.2, here we give the proofs 4.2:

Proof. 4.2:

At time T, the CDs's contingent payoff is:

$$Z_T^{CDs}=V_T-Max(K-V_T,0)=\begin{cases}K-V_T & \text{if } V_T<K\\ 0 & \text{if } V_T<K\end{cases}$$

By inserting in these formulas the prices of security, the equilibrium price of the CDs:

$$CDs_t=\frac{E_t[(1+\xi_t)A_T^{-1}(K-V_T)^+]}{E_t[(1+\xi_t)A_T^{-1}]}$$

The numerator can be decomposed as follows:

$$E_t[(1+\xi_t)A_T^{-1}(K-V_T)^+]=E_t[A_T^{-1}(K-V_T)1_{V_T\le K}\mid F_t]+E_t[\xi_tA_T^{-1}(K-V_T)1_{V_T\le K}\mid F_t]$$

Under the filter F_t , the following two expressions have equivalence relation $V_T\le K\Leftrightarrow B_{T-t}\le(lnK-\mu T)/\sigma-B_t+\frac{1}{2}\sigma T$.

By inserting in the inequality, the first part in the numerator follows:

$$\begin{aligned}&E_t[A_T^{-1}(K-V_T)1_{V_T\le K}\mid F_t]\\&=Ke^{-\mu T-\sigma B_t+\sigma^2T-\frac{1}{2}\sigma^2t}\int_{-\infty}^{c}\frac{1}{\sqrt{2\pi(T-t)}}e^{-\frac{1}{2(T-t)}[B(T-t)+(T-t)\sigma]^2}dB_{T-t}\\&-\int_{-\infty}^{c}\frac{1}{\sqrt{2\pi(T-t)}}e^{-\frac{1}{2(T-t)}B(T-t)^2}dB_{T-t}\\&=kV_t^*\Phi(d_1)-\Phi(d_2)\end{aligned}$$

Where,$d_1 = c/\sqrt{T-t} + \sigma\sqrt{T-t}$,$d_2 = d_1 - \sigma\sqrt{T-t}$,$c = (lnk - \mu T)/\sigma - B_t + \frac{1}{2}\sigma T$.

In the same way, the second part of the numerator follows:

$$E_t[\xi_t A_T^{-1}(K - V_T) 1_{V_T \leq K} \mid F_t] = KV_t^* g\Phi(d_1 - \sigma\eta\sqrt{T_t}) - ge^{(T-t)\sigma^2\eta}\Phi(d_2 - \sigma\eta\sqrt{T_t})$$

The denominator $E_t[(1+\xi_T)A_T^{-1}]$ is detailed calculated in proof of proposition, finally the equilibrium value of the CDs is :

$$CDs_t = K\frac{\Phi(d_1) + g\Phi(d_1 - \sigma\eta\sqrt{T-t})}{1+g} - V_t^* \frac{\Phi(d_2) + ge^{\eta\sigma^2(T-t)}\Phi(d_2 - \sigma\eta\sqrt{T-t})}{1+g}$$

Similarly ,the limiting prices of CDs with rational and irrational belief are $CDs_t^* = K\Phi(d_1) - V_t^*\Phi(d_2)$ and $S_t^* = V_t^*\Phi(-d_2) - k\Phi(-d_1)$ respectively. $\square$