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Segregation according to household size in a monocentric city

by

Theis Theisen

Abstract

Over the last two centuries, household size has decreased considerably. Within a theoretical model I investigate the relationship between household size and the structure and size of cities. Household utility is assumed to depend on household size, in addition to the consumption of housing and a numeraire good. This basic building block is combined with a Muth-type urban model. The model is used for examining the impact of household size on the sorting of households according to household size, the geographical extension of the city, household utility, forms of housing etc. These issues have previously received some attention in empirical studies, but I am not aware of theoretical examinations.

Contact details: Adress: Theis Theisen, Department of Economics and Business Administration, Faculty of Economics and Social Sciences, University of Agder, Servicebox 422, 4604 Kristiansand, Norway. E-mail: Theis.Theisen@uia.no. Phone: +47-38141526, Fax: +47-38141061.

1. Introduction

In industrialized countries, household size has over the last two centuries decreased considerably. This demographic transition has taken several forms. In an early phase, multi-generation households split up in separate households, and the number of children in each family declined. In a later phase, families split up in single-parent and single-person households, young people lived as singles for a longer period than was usual some decades ago, and the number of elderly living as singles also increased. In the present paper, the focus is mainly on the third of these phases. A few numbers may illustrate how dramatic these changes have been. In 1950, about 15 per cent of the Norwegian households consisted of only one person. By 2001 this share had increased to 38 per cent. Moreover, in the largest city in the country, Oslo, the majority of households today contain only one individual.

The effects that arise when multi-person households replace single-person households are quite complex, both at the household level and at the social level. At the household level, a single-person household usually will have a lower income than a larger household. If there are economies of scale in household consumption, individuals in single-person households will presumably also obtain a lower utility level than they could enjoy in a larger household. These differences are likely to result in different consumption patterns. In particular, in a city containing many single-person households a larger number of housing units will be demanded. Presumably, single-person households will also occupy smaller dwellings than multi-person households. This will have a strong impact on the land market, population density, and the geographical extension of the city. One would also expect that households of different size would settle in separate segments of the city. This raises the question of who would live in the central parts and who at the outskirts. Another related issue of some interest is which group will live in apartments in the central parts of the city and who will live in low-density detached dwellings. To the best of my knowledge, these issues have not previously been subjected to a thorough theoretical examination, despite the fact that they have a profound impact on many aspects of people's lives. In the present paper I therefore examine the impact on the urban housing sector of the increasing number of single-person households in urban areas.

In empirical research, the impact of household size on the urban housing sector has received some attention. (Add some references here) Skaburskis (1999) finds that a reduction in household size leads to a drop in the demand for low-density housing. A better theoretical understanding of such relationships is however needed, inter alia in order to improve the specification of empirical models and facilitate the interpretation of the results. In addition, a better theoretical grasp of the role of household size in the urban housing sector is needed to guide policy decisions made by local governments. For instance, some local governments pursue an active housing policy in order to increase the number of multi-person households with children that live in the central parts of cities. Presumably, the analysis in the present paper will prove to be useful in judging whether such policies are warranted.

In the next section, we set up the baseline model of household choice between housing and a numeraire good. The baseline model is the same as the one used by Brueckner (1987), but we extend the model by introducing households of different size, and derive the condition that determine whether single-person households will reside in the central parts of the city or in the outskirts. In Section 3, we set out a very simple model of the supply side of the housing market. In Section 4 we establish the conditions that must be satisfied in order for the housing market to be in equilibrium. Section 5 contains a comparative-static analysis of how a change in the share of households of different size will affect the equilibrium. Section 6 summarize the main results and provides some ideas for further research.

2. Spatial segregation of single-person and multi-person households

The baseline model

We consider a mono-centric city inhabited by individuals with identical preferences. Some individuals live in single-person households, while others live in two-person households. Households derive utility from a hicksian composite good (c_n) and from the floor space (q_n) of their dwelling, where n indicates household size. Let $v = v(c_n, q_n; n)$ be the household utility function. Since there may be economies of scale in household consumption, in particular for housing, household size enters the household utility function. The utility function is assumed to be strictly quasi-concave in c_n and q_n. The members of the household work a fixed number of hours at a fixed wage rate. This gives the household an exogenous income, y. The household spends its income on the composite good, on renting floor space, and on commuting to work in the (spaceless) centre of the city. Total costs of commuting amounts to $t_n x$, where x is commuting distance, and t_n is the cost of commuting one kilometre (roundtrip) for all members of a household of size n. When the composite good is chosen as the numeraire, the budget constraint takes the form $y_n = c_n + p(x)q_n + t_n x$, where $p(x)$ is the rental cost per square metre of floor space located at distance x from the city centre. Substituting from the budget constraint into the utility function, the utility maximization problem of a n-person household living at distance x from the city centre takes the form:

$$\underbrace{Max}_{q_n} v(y_n - p(x)q_n - t_n x, q_n; n) \tag{1}$$

This yields the first order condition, where v_n^c and v_n^q denote partial derivates of the utility function:

$$\frac{v_n^q(y_n - t_n x - pq_n, q_n; n)}{v_n^c(y_n - t_n x - pq_n, q_n; n)} = p\,. \tag{2}$$

Since individual preferences by assumption are identical, all households of a given size will have identical preferences. We also assume that all households of a given size have the same income and costs of transportation. Under these assumptions, households of a given size living at different distances from the city centre must obtain the same utility. Households of different size will, however, in general differ in income and costs of transportation, and will therefore enjoy different levels of utility. Hence, we have the restriction:

$$v(y_n - p(x)q_n - t_n x, q_n; n) = \bar{v}_n \qquad n \in [1,2], \tag{3}$$

where the bar on the r.h.s. variable indicates that there is a common utility level for all households of a given size. We return in Section 4 to how these utility levels are determined. Differentiating Eq. (3) w.r.t. x yields the rent gradient:

$$\frac{\partial p_n}{\partial x} = \frac{-t_n}{q_n} \qquad n \in [1,2]. \tag{4}$$

Eq. (4) tells us that the rent gradient for floor space in equilibrium drops as one move away from the city centre. Hence, households with high commuting costs are in equilibrium compensated through a lower rent on floor space, so that all households of a given size will enjoy the same utility, irrespective of where in the city they live.

Totally differentiating Eqs. (2) and (3) w.r.t. x, y_n, t_n, and $\bar{v}_n$, we can find the impact of these variables on the price of floor space and the demand for floor space. Since a complete accord of this can be found in Brueckner (1987), we here just summarize these results in Table 1 below, where $\omega_n < 0$ is the slope of the hicksian (constant utility) demand curve, and $MRS_n = -v_n^q / v_n^c$ is the marginal rate of substitution between the two goods in the utility function

Table 1. The impact on floor space and price of floor space of changes in x, y_n, t_n, and $\bar{v}_n$

	Impact of an increase in:			
	x	y_n	t_n	$\bar{v}_n$
Impact on p	$\frac{\partial p_n}{\partial x} = \frac{-t_n}{q_n} < 0$	$\frac{\partial p_n}{\partial y_n} = \frac{1}{q_n} > 0$	$\frac{\partial p_n}{\partial t_n} = \frac{-x}{q_n} < 0$	$\frac{\partial p_n}{\partial \bar{v}_n} = \frac{-1}{q_n v_n^c} < 0$
Impact on q	$\frac{\partial q_n}{\partial x} = \omega_n \frac{\partial p_n}{\partial x} > 0$	$\frac{\partial q_n}{\partial y_n} = \omega_n \frac{\partial p_n}{\partial y_n} < 0$	$\frac{\partial q_n}{\partial t_n} = \omega_n \frac{\partial p_n}{\partial t_n} < 0$	$\frac{\partial q_n}{\partial \bar{v}_n} = \omega_n \left[\frac{\partial p_n}{\partial v_n} - \frac{\partial MRS_n}{\partial c_n} \frac{1}{v_n^c} \right] > 0$

Modelling spatial segregation according to household size

As indicated by Eq. (4), the rent gradient will in general be different for households of different size. Since floor space at a given location will be rented out to the highest bidder, the household type with the steepest rent gradient will occupy the central parts of the city. [1] Consequently, if single-person households have a steeper rent gradient than two-person households, single-person households will in equilibrium bid up rents in the central parts of the city, and occupy the central part, while two-person households will be willing to pay the highest rents in the outer parts, and will occupy this part. On the contrary, if single-person household have a less steep rent gradient than two-person households, single-person households will in equilibrium occupy the outer part of the city, and two-person households the inner part. In order to examine which of these two cases apply, consider the situation at distance $\tilde{x}$ from the city centre, where we find the borderline between the areas occupied by the two household types. At this location the rent-functions of two types of households cross, and they will both have to pay the same rent per unit of floor space. Initially, let us also assume that the two households have the same income, and that their transportation costs also are identical (these initial assumptions will be modified below). Preferences are, however, assumed to differ between household types as stated by:

Assumption 1: Let $MRS_n = -v_n^q / v_n^c$ *be the marginal rate of substitution between floor space and the numeraire good for a household of size* $n \in \{1,2,\}$. *Preferences satisfy the restriction* $MRS_2 > MRS_1$.

[1] For instance, Brueckner (1977) demonstrates that high-income households will settle on large floor space in the outskirts of the city, while low-income households will occupy the central parts of the city. In a similar vein, Solow (1971) demonstrates that business activity will take place in the central parts of the city, while the recidential areas will occupy the less central parts.

In other words, Assumption 1 restricts preferences to be single-crossing. (Ad reference) In the present context an important rationale behind single-crossing preferences is that there is likely to be economies of scale in the consumption of housing, while this is likely to be considerably less prevalent for the numeraire good. In Figure 1 the single-crossing preferences are illustrated. At point A, v_2 is an indifference curve for a two-person household, while v_1 is an indifference curve for a single-person household. If the two households have the common income net of transportation costs represented by the budget constraint FG, the optimum for the two-person household is at point A, while point B will maximize the utility of the single-person household. Hence, if the two households have the same incomes and costs of transportation, the single-person household will with the type of single-crossing preferences stated in Assumption 1 consume less floor space than the two-person household. At distance $\tilde{x}$ from the city centre a single-person household will then have a steeper rent gradient than a two-person household. Consequently, single-person households will then occupy the inner part of the city.

The assumption that households containing one and two persons have the same income and transportation costs is highly unrealistic. Hence, in the sequel the analysis will be based on the following assumption:

Assumption 2: Each adult person earns the same income, y.

Assumption 2 implies that income per capita in the city will be independent of city population and of how individuals group themselves together in households. At least as a first approximation, this seems reasonable. Moreover, since Wheaton (1974), Brueckner (1987) and many other have found that average per capita income has a profound impact on the equilibrium of the urban housing sector, we want to neutralize this source of difference between cities. Assumption 2 serves this purpose.

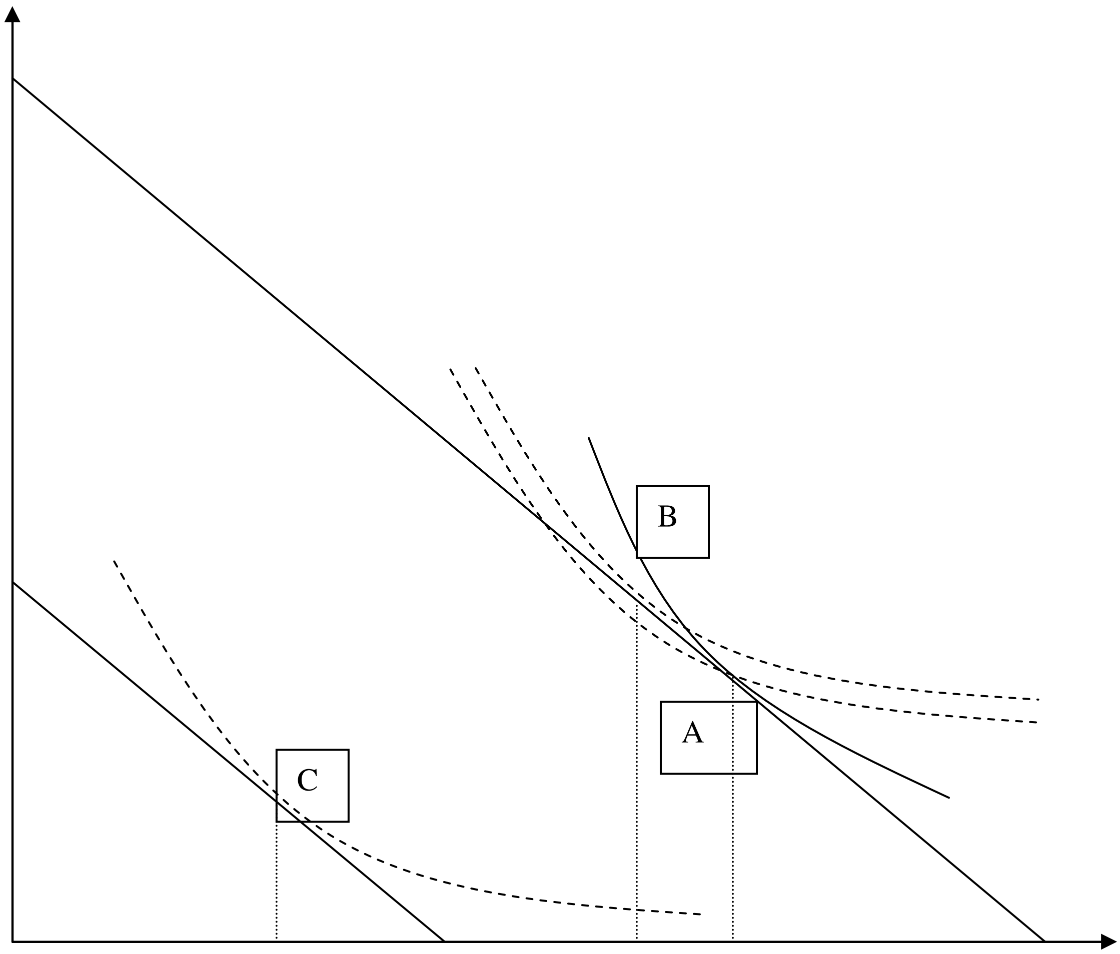

Figure 1 Single-crossing preferences for households of different size.

In order to be able to make precise inference of how the income difference between households of different size affects the consumption of housing, we also incur the following assumption, which is frequently made in different variants of the mono-centric urban model:

Assumption 3: Housing is a normal good

Next. it is reasonable to assume that costs of transportation differ systematically between households of different size. If each (adult) individual in the household holds a full job, and if all costs of transportation to the city-centre were related to job-commuting, it would be reasonable to assume $t_2 = 2t_1$. Households incur, however, transportation costs also when they travel to the city centre for shopping, or for visiting cultural amenities, restaurants, etc. When measured on a per person basis, such transportation costs are likely to be much higher for individuals living as singles than for individuals in two-person households. One reason for this is that one individual in a two-person households often may do shopping for the whole household. Another reason is that a single-person household is likely to incur more commuting costs than a two-person household in order to obtain social contact with other individuals at restaurants, cultural amenities, etc. For a two-person household much of the social contact is obtained within the household, without any costs of transportation.[2] Based on these arguments, we assume:

Assumption 4: The costs of transportation for households of size 1 and 2 are related as follows $t_1 = \alpha t_2$, *where the parameter* $\alpha \in (0.5,1)$.

Assumptions 2 and 4 combined implies that income net of transportation costs for a single-person household living at distance $\tilde{x}$ from the city centre, will be less than half of what it is for a two-person household at the same location. The budget constraint of a single-person household is in Figure 1 shown as the line IJ.

[2] At the cost of a substantially more complex modell it would be possible to include social contact in the utility function and to let the consumer choose endogenously the volume of this good. The simpler approach taken in the present paper correspons well, however, to the assumption that work participation as well as working hours and number of commuting trips are tken as exogenous.

Under Assumptions 1, 2, 3, and 4 the optimum of a single-person household living at distance $\tilde{x}$ from the city centre will be at point C in Figure 1, while it for a two-person household will be at A. From Figure 1 we can then conclude that the consumption of floor space for a single-person household compared to a two-person household, both living at distance $\tilde{x}$ from the city centre, is affected negatively by the difference in preferences as well as by the drop in income net of transportation costs.

The analysis so far tells us that a single-person household living at distance $\tilde{x}$ from the city centre under reasonable assumptions will have a lower consumption of floor space than a two-person household. However, since the single-person household also has lower transportation costs, additional restrictions are needed in order to determine which household type will have the steepest rent gradient at $\tilde{x}$. For this purpose, let us rewrite the rent-gradient for a single-person household as:

$$\frac{\partial p_1}{\partial x} = \left(\frac{-t_2}{q_2}\right)\underbrace{\left(\frac{\alpha}{(1-\beta_1)(1-\beta_2)}\right)}_{\gamma}, \qquad (5)$$

where $\beta_1 \in [0,1)$ is the relative reduction in demand due to different preferences for a household of size 1 compared to a household of size 2, i.e. the move from A to B in Figure 1. Next, $\beta_2 \in [0,1)$ is the relative drop in demand due to Assumption 2, i.e. that a single-person household has a lower income than a two-person household. If we in Eq. (5) have $\gamma > 1$, a single-person household will have a steeper rent gradient than a two-person household. This yields:

Proposition 1. If $\alpha > (1-\beta_1)(1-\beta_2)$*, single-person households have at* $\tilde{x}$ *a steeper rent gradient than two-person households. When this condition is fulfilled, single person households will occupy the dwellings closer to the city centre than* $\tilde{x}$*,while two-person households will live further from the city centre than* $\tilde{x}$*.*

In order to assess the implications of the condition in Proposition 1, let us plug in the borderline values $\alpha = 0.5$ and $\beta_1 = 0$. We then obtain the condition $\beta_2 > 0.5$. Combined with our assumption of how household income changes when a two-person household is split in two, $\beta_2 > 0.5$ implies that the income elasticity, E^q, in the demand for floor space must not be less than 1. Empirical estimates of this elasticity often lie around 1. Notice, however, that this result is calculated on the very extreme assumptions that $\alpha = 0.5$ and $\beta_1 = 0$. If we plug in the more realistic assumptions $\alpha = 0.6$ and $\beta_1 = 0.2$, the condition in proposition 2 will be satisfied for $\beta_2 > 0.25$, which is satisfied for $E^q \geq 0.50$. Since vast majority of estimated income elasticities for housing are higher than this, we conclude that the condition stated in Proposition 1 is likely to be satisfied, and that single-person households therefore in equilibrium is likely to outbid two-person households for the dwellings closer to the city centre than $\tilde{x}$. This situation is illustrated in Figure 2. The analysis in the sequel is based on this.

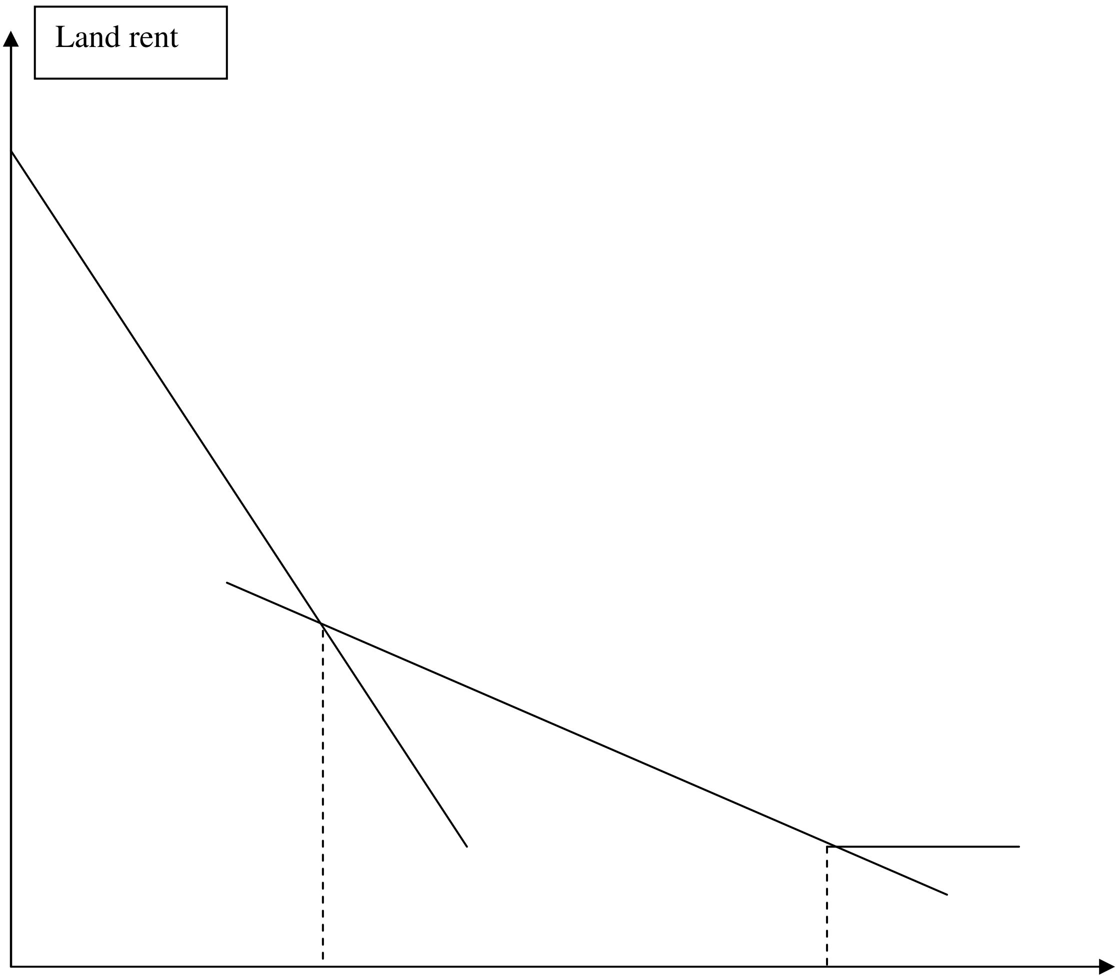

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Figure 2 Spatial segregation of single-person and two-person households

3. Segregation of households in different building types

The supply side of the housing market is represented by entrepreneurs who rent dwellings to households. The supply-side model has previously been set out by Brueckner (1987). Housing entrepreneurs produce housing, measured in square meters of floor space, from land (l) and physical capital (K). The housing production function, $H(K,l)$, is assumed to be concave, and to exhibit constant returns to scale. An entrepreneur's revenue from renting out the dwellings contained in the building sitting on a piece of land under his command is $pH(K,l)$. In order to make the analysis tractable, we assume that housing entrepreneurs rent land and physical capital from landlords and capital owners living outside the city under consideration. Let r denote the endogenously determined, spatially variable, rent per square meter of land, and let i denote the exogenously given, spatially invariant, rent per unit of physical capital. The entrepreneur earns a profit $(pH(K,l) - iK - rl)$ from renting out the structure, including the land affiliated with it, to households. With constant returns to scale the profit can also be written $l(pH(K/l,1) - i\,K/l - r)$, and introducing the capital-land ratio $(S = K/l)$, which hereafter is denoted the structural density, the profit expression can be simplified to:

$$l(ph(S) - iS - r) \tag{6}$$

In this equation, $h(S) \equiv H(S,1)$ measures floor space per square-meter of land. We assume $h'(S) \equiv H_1(S,1) > 0$, $h''(S) \equiv H_{11}(S,1) < 0$, where $H_1(S,1) > 0$ and $H_{11}(S,1) < 0$ are the first and second order derivates of the production function $H(S,1)$ w.r.t its first argument. For a lot at a specific location, the entrepreneur maximizes profit through choosing the optimal structural density (building height), taking the land-rent at the specific location as exogenously given. This yields the first-order condition stating that in optimum the marginal revenue from renting out the building structure must be equal to the marginal cost of physical capital:

$$ph'(S) = i\,. \tag{7}$$

With constant returns to scale, producer profit must in equilibrium also be equal to zero. This yields the condition:

$$ph(S) - iS = r, \tag{8}$$

which tells that the profit net of the rent that the entrepreneur must pay for the physical capital embedded in the building structure in optimum must be equal to the land rent. The land-rent at each location must in equilibrium adjust to the levels that fulfil condition (8).

In Section 2 we found that the price per square meter floor space is a function of $\varphi = x, y_n, t_n, \bar{v}_n$. Taken together with Equations (7) and (8) this means that structural density (S) and land rent (r) will be functions of the same variables, plus the rent (i) for a unit of physical capital (which we assume exogenous and fixed throughout the analysis). Totally differentiating Eqs. (7) and (8) w.r.t. $\varphi = x, y_n, t_n, \bar{v}_n$, and solving for the impact of these variables on structural density and land rent yields:

$$\frac{\partial r}{\partial \varphi} = h \frac{\partial p}{\partial \varphi} \qquad (\varphi = x, y_n, t_n, \bar{v}_n) \tag{9}$$

$$\frac{\partial S}{\partial \varphi} = -\frac{h'}{ph''} \frac{\partial p}{\partial \varphi} \qquad (\varphi = x, y_n, t_n, \bar{v}_n) \tag{10}$$

Since $h' > 0$, and $h'' < 0$, we obtain by using the results for $\partial p / \partial \varphi$ from Table 1 in Section 2:

$$\frac{\partial r}{\partial x} < 0, \ \frac{\partial S}{\partial x} < 0 \tag{11}$$

Hence, the land rent and buildings are lower the further one come from the city centre.

Comment on relationship with household size in this Section or in the next?

4. Equilibrium conditions for a city with two household types

Let M_1 be the number of single-person households, while M_2 is the number of two-person households. We consider a city with an exogenously given population N, i.e. the closed city case. Hence, the total population of the city is $N = M_1 + 2M_2$. We want to examine how a decrease in the number of two-person households which is matched by an increase in the number of single-person households, conditional on the population of the city being fixed, affects the equilibrium of the city. That is, we will be interested in the effect on the geographical extension of the city, the distance from the city centre to the borderline between the areas occupied by the two households types, the rent level, the floor size of dwellings, structural density, population density, and the utility of each of the two household types.

In order to examine these issues we have to add three equilibrium conditions to the model set out in Sections 2 and 3. First, the two-person households living at the border of the city, $\bar{x}$, must, through the price they pay for renting floor space at that location, be willing to pay a land-rent that matches the rent that landlords can earn from renting out the land to agriculture (assuming that agriculture is the alternative land-use that would be willing to pay the highest rent). Denoting the exogenously given land rent in agriculture by r_A, this gives the condition:

$$r(\bar{x}, y_2, t_2, \bar{v}_2) = r_A . \tag{12}$$

Since we in Section 3 found that land rent declines as one move away from the city centre, two-person households will, when the condition in Proposition 1 is satisfied, outbid agricultural land use between $\bar{x}$ and $\tilde{x}$.

The second equilibrium condition states that at the borderline of the area occupied by single-person households and two-person households, the two types of households must be willing to pay exactly the same rent for a unit of land. This condition is formalized as:

$$r(\tilde{x}, y_1, t_1, u_1) = r(\tilde{x}, y_2, t_2, u_2). \tag{13}$$

Again, when the condition in Proposition 1 is satisfied, single-person households will outbid two-person households closer to the city centre than $\tilde{x}$, while the opposite will be the case between $\tilde{x}$ and $\bar{x}$.

Next, let the housing density (number of households per unit of land) at distance x from the city centre be $D(x, y_n, t_n, \bar{v}_n)$. The two last equilibrium conditions require that each of the population segments (households of size 1 and 2) must fit within the areas of the city that they occupy. When θ denotes the number of radians that are available for housing, at each $x \leq \bar{x}$, these conditions take the form:

$$\int_0^{\tilde{x}} x\theta D(x, y_1, t_1, \bar{v}_1)dx = M_1, \tag{14}$$

$$\int_{\dddot{x}}^{\bar{x}} x\theta D(x, y_2, t_2, \bar{v}_2)dx = M_2. \tag{15}$$

Eqs. (14) and (15) can be modified by writing housing density as follows:

$$D = \begin{cases} h/q = -\left(\partial r_1/\partial q_1\right)/t_1 & \text{for single - person households} \\ h/q = -\left(\partial r_2/\partial q_2\right)/t_2 & \text{for two - person households} \end{cases} . \tag{16}$$

Substituting this into Eqs. (14) and (15) we obtain:

$$-\int_0^{\tilde{x}} x\left(\partial r_1/\partial x\right)dx = t_1 M_1/\theta , \tag{17}$$

$$-\int_{\ddot{x}}^{\bar{x}} x\left(\partial r_2/\partial x\right)dx = t_2\, M_2/\theta . \tag{18}$$

Integrating by parts on the l.h.s. of Eqs. (17) and (18) we obtain:

$$-\tilde{r}\tilde{x} + \int_0^{\tilde{x}} r dx = t_1 M_1/\theta , \tag{19}$$

$$-\bar{x} r_A + \tilde{r}\tilde{x} + \int_{\ddot{x}}^{\bar{x}} r dx = t_2\, M_2/\theta , \tag{20}$$

where $\tilde{r}$ is the land rent at distance $\tilde{x}$ from the city centre.

The four Equations (12), (13), (19), and (20) determinine the following four endogenous variables: The extension of the city, $(\bar{x})$, the distance between the city centre and the borderline $(\tilde{x})$ between the areas where the two population segments live, and the utility levels of the two population segments $(\bar{v}_1, \bar{v}_2)$.

5. Comparative static analysis of a city with two household types

Totally differentiating Eqs. (12), (13), (19), and (20) w.r.t. $\chi = M_1, M_2, \alpha, \beta$, taking costs of transportation, i, and r_A as fixed, and using the results from Sections 2 and 3, we can find the impact on $\bar{x}, \tilde{x}, \bar{v}_1$, and $\bar{v}_2$. First, differentiating Eqs. (12) and (13) we obtain:

$$\frac{\partial \bar{x}}{\partial \chi} = -\frac{\dfrac{\partial r_2}{\partial \bar{v}_2}\dfrac{\partial \bar{v}_2}{\partial \chi}}{\dfrac{\partial r_2}{\partial \bar{x}}}, \tag{21}$$

$$\frac{\partial \tilde{x}}{\partial \chi} = \frac{-\dfrac{\partial r_1}{\partial \bar{v}_1}\dfrac{\partial \bar{v}_1}{\partial \chi} + \dfrac{\partial r_2}{\partial \bar{v}_2}\dfrac{\partial \bar{v}_2}{\partial \chi}}{\left(\dfrac{\partial r_1}{\partial \tilde{x}} - \dfrac{\partial r_2}{\partial \tilde{x}}\right)}. \tag{22}$$

In these equations, the subscripts on the land-rent functions indicate household type. Next, differentiating Eqs. (19) and (20) w.r.t. $\chi = M_1, M_2, \alpha, \beta$ and substituting from Eq. (22) in the two resultant equations, we obtain:

$$\begin{bmatrix} -\begin{bmatrix} \left(\dfrac{\partial \tilde{r}_1}{\partial \bar{v}_1}\tilde{x} - \displaystyle\int_0^{\tilde{x}} \dfrac{\partial r_1}{\partial \bar{v}_1}dx\right)\left(\dfrac{\partial r_1}{\partial \tilde{x}} - \dfrac{\partial r_2}{\partial \tilde{x}}\right) \\ -\left(\dfrac{\partial \tilde{r}_1}{\partial \tilde{x}}\tilde{x} + r_1(0) - \tilde{r}_1\right)\dfrac{\partial \tilde{r}_1}{\partial \bar{v}_1} \end{bmatrix} & -\left[\left(\dfrac{\partial \tilde{r}_1}{\partial \tilde{x}}\tilde{x} + r_1(0) - \tilde{r}_1\right)\dfrac{\partial r_2}{\partial \bar{v}_2}\right] \\ -\left[\dfrac{\partial r_1}{\partial \bar{v}_1}\left(\dfrac{\partial r_2}{\partial \tilde{x}}\tilde{x} + r_A - \tilde{r}_2\right)\right] & \begin{bmatrix} \left(\dfrac{\partial r_2}{\partial \bar{v}_2}\tilde{x} + \displaystyle\int_{\tilde{x}}^{\bar{x}} \dfrac{\partial r_2}{\partial \bar{v}_2}dx\right)\left(\dfrac{\partial r_1}{\partial \tilde{x}} - \dfrac{\partial r_2}{\partial \tilde{x}}\right) \\ +\dfrac{\partial r_2}{\partial \bar{v}_2}\left(\dfrac{\partial r_2}{\partial \tilde{x}}\tilde{x} + r_A - \tilde{r}_2\right) \end{bmatrix} \end{bmatrix} \begin{bmatrix} \dfrac{\partial \bar{v}_1}{\partial \chi} \\ \dfrac{\partial \bar{v}_2}{\partial \chi} \end{bmatrix} = \begin{bmatrix} \left(\dfrac{\partial r_1}{\partial \tilde{x}} - \dfrac{\partial r_2}{\partial \tilde{x}}\right)\left(\dfrac{\partial t_1 M_1}{\partial \chi}\right) \\ \dfrac{\left(\dfrac{\partial r_1}{\partial \tilde{x}} - \dfrac{\partial r_2}{\partial \tilde{x}}\right)}{2}\dfrac{\partial (t_2 M_2)}{\partial \chi} \end{bmatrix} \tag{23}$$

From this we can find the impact on the utility level of the two household groups of an increase in the share of single-person households. Once the impact on the utility levels are found, Eqs. (21) and (22) give the effect on the border of the city and the border between the areas occupied by the two households types. Next we can find the impact on rents, building heights, etc. The derivation of these results, which is very much inspired of Brueckner (1983) and Wheaton (1976), is quite technical, and is therefore relegated to the appendix which will be included in the final version of the paper. The results can be summarized as follows:

Proposition 2: When the number of two-person households drops, and the members of these households form new single-person households, the following changes will occur: (I) the utilities of both household types will decline, (2) the city border will be pushed outwards, (3) the border between the inner part of the city occupied by single-person households and the outer part occupied by two-person households will be pushed outwards, (4) land rent and the rent of floor space will increase at each location, (5) bilding height will increase at each location, and (6) housing density and population density will increase at each location.

6. Concluding remarks

Within a framework where all *individuals* have identical preferences, but where economies of scale gives rise to different single-crossing preference maps for single-person and two-person households, we demonstrated that single-person households will occupy the central parts of cities, while two-person households will live in the outer parts. It is noteworthy that these results basically follow from economies of scale in households and competition for land.

Single-person households will enjoy a lower utility level than two-person households. Hence, single person households have a strong incentive to form two-person households, but this is counteracted by dissolution of two-person households. The household formation and dissolution processes were taken as exogenous in our analysis. We studied the consequences of a net increase in the share of households containing only one person, in a situation where the city's total population was fixed, and where also per capita income was fixed. We found that this would lead to a decline in the utilities of both household types, and therefore also a decline in the welfare of city's inhabitants. Due to the increase in the number of households, the city border will be pushed outwards. Likewise, the border between the inner part of the city occupied by single-person households and the outer part occupied by two-person households will be pushed outwards. Through increased competition for land the land rent will increase at each location, and this in turn induces an increase in building height at each location.

References

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